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BIFURCATION PROPERTIES OF SEMILINEAR ELLIPTIC EQUATIONS IN $\mathbf{R}^n$

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Abstract. We study the bifurcation properties of the semilinear equation

$$\Delta u + \lambda f(x)(u + h(u)) = 0, \quad x \in \mathbf{R}^n,$$

where $h : \mathbf{R} \to \mathbf{R}$ is a bounded Hölder continuous function satisfying

$$\lim_{\xi \to 0+} \frac{h(\xi)}{\xi} \equiv a > 0,$$

and $f : \mathbf{R}^n \to \mathbf{R}$ is a positive asymptotically radial function satisfying $\int_{\mathbf{R}^n} f(x)\, dx < \infty$. We show the existence of a connected branch of positive solutions, u satisfying $|x|^{n-2}u(x) \to c > 0$ as $|x| \to \infty$. Such a branch bifurcates from infinity at λ_1, where λ_1 is the principal eigenvalue of the linear equation $\Delta u + \lambda f(x)u = 0$, and from the trivial solutions at $\lambda_0 = \lambda_1/(1+a)$. We use the Leray-Schauder degree theory applied to the corresponding operator equations, and a global bifurcation result of Rabinowitz.

0. Introduction. Let $f \in C^{\alpha}_{\text{loc}}(\mathbf{R}^n, \mathbf{R})$ for some $0 < \alpha < 1$ and $n \geq 3$. Suppose there exist functions p and P, both in $C(\mathbf{R}^+, \mathbf{R}^+)$, with $0 < p(|x|) \leq f(x) \leq P(|x|)$ for all $x \in \mathbf{R}^n$, and

$$\int_{\mathbf{R}^n} |x|^{2-n} P(|x|)\, dx < \infty. \tag{1}$$

Let $h : \mathbf{R} \to \mathbf{R}^+$ be a bounded Hölder continuous function with exponent α satisfying $h(0) = 0$, and $\lim_{\xi \to 0+} \frac{h(\xi)}{\xi} \equiv a > 0$. In a recent paper [4] the authors have shown that if h is nondecreasing and a is large enough, there exists an interval of λ values for which the semilinear eigenvalue problem

$$\Delta u + \lambda f(x)(u + h(u)) = 0 \quad x \in \mathbf{R}^n \tag{2}$$

has a bounded positive solution u satisfying $\lim_{|x| \to \infty} |x|^{n-2}u(x) = c > 0$. In fact, if we let

$$l_1 = \frac{n-2}{\int_0^\infty \rho P(\rho)\, d\rho} \quad \text{and} \quad l_2 = \frac{n-2}{\int_0^1 \rho^{n-1} p(\rho)\, d\rho},$$

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