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GENERALIZED MOTION OF HYPERSURFACES WHOSE GROWTH SPEED DEPENDS SUPERLINEARLY ON THE CURVATURE TENSOR

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Abstract. We prove a comparison principle for viscosity solution with finite speed for its level set, which solves degenerate parabolic equations with discontinuity. We also prove the (global) existence of solution in the class of viscosity solution with finite speed for the initial value problem. Our comparison and existence results yield a unique global-in-time generalized solution to interface evolution equations whose speed grows superlinearly in curvature tensors.

1. Introduction. Let $\Gamma(t)$ be an interface bounding the whole space R^N ($N \geq 2$) into two phases at time $t \geq 0$. To write down the equation of $\Gamma(t)$ we temporarily assume that $\Gamma(t)$ is the smooth boundary of the open set $D(t)$. The evolution of $\Gamma(t)$ that we consider here depends locally on its normal vector field and curvature tensors.

Let $\vec{n} = \vec{n}(t,x)$ denote the unit exterior normal vector field to $\Gamma(t) = \partial D(t)$ at $x \in \Gamma(t)$. It is convenient to extend $\vec{n}$ to a vector field, still denoted by $\vec{n}$, on a tubular neighborhood of $\Gamma(t)$ such that $\vec{n}$ is constant in the normal direction of $\Gamma(t)$. Let $V = V(t,x)$ denote the growth speed of $\Gamma(t)$ at $x \in \Gamma(t)$ in the exterior normal direction. In this paper, as a continuation of [5] and [14], we study the evolution equation of form

$$V = f(\vec{n}, \nabla\vec{n}) \quad \text{on } \Gamma(t), \quad t > 0. \tag{1.1a}$$

Here f is a given continuous function and ∇ stands for spatial derivatives. We are interested in constructing global-in-time solutions (family) $\{\Gamma(t)\}_{t\geq 0}$ to the evolution equation (1.1a) under the initial condition

$$\Gamma(t)|_{t=0} = \Gamma_0, \tag{1.1b}$$

where Γ_0 is an arbitrary given (compact) initial interface.

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