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BLOW-UP FOR SEMILINEAR WAVE EQUATIONS WITH INITIAL DATA OF SLOW DECAY IN LOW SPACE DIMENSIONS

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Abstract. We are concerned with the blow-up of classical solutions to the Cauchy problems for $\Box u = |u_t|^p$ in $\mathbb{R}^n \times [0,\infty)$, $1 \le n \le 3$. For this subject, the effect of the power p has been studied extensively, provided the initial data are compactly supported. However, the decay rate of the initial data also has an important role as well as the power p. Indeed, the slow decay of them causes blow-up in finite-time for any $p > 1$.

1. Introduction and statement of results. This paper is concerned with finite-time blow-up of solutions to the Cauchy problems:

$$\begin{cases} u_{tt} - \Delta u = F(u_t) \quad \text{in} \quad \mathbb{R}^n \times [0,\infty), \\ u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x) \quad \text{for} \quad x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where u is a scalar unknown, $1 \le n \le 3$, $F(\lambda) \in C^2(\mathbb{R})$, $\phi(x) \in C^{[n/2]+2}(\mathbb{R}^n)$ and $\psi(x) \in C^{[n/2]+1}(\mathbb{R}^n)$. For the nonlinearity, we assume that there exist constants $p > 1$ and $A > 0$ such that

$$F(\lambda) \ge 2A|\lambda|^p \quad \text{for} \quad \lambda \in \mathbb{R}. \tag{1.2}$$

Our main goal is to find a class of initial data for which solutions to the Cauchy problem (1.1) blow up in finite-time for any $p > 1$. The basis of our method is to convert the problem (1.1) into a first order ordinary differential inequality along a characteristic ray (2.6) below, by employing K. Masuda's lemma. (See [14], also Lemma 2.1 below).

Concerning the global behavior of the solutions to the Cauchy problem for nonlinear wave equation, many results have been established over past years; for instance [1]-[27], ... Let us mention here some of them which are closely related to our problem. To begin with, we mention a blow-up result established by R.T. Glassey in [4] for either the Cauchy problem (1.1) or the Cauchy problem

$$\begin{cases} u_{tt} - \Delta u = F(u) \quad \text{in} \quad \mathbb{R}^n \times [0,\infty), \\ u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x) \quad \text{for} \quad x \in \mathbb{R}^n. \end{cases} \tag{1.3}$$

His results imply that for the large initial data, no global in time solution exists. Therefore in what follows, we consider the 'small' initial data described as

$$\phi(x) = \varepsilon f(x), \quad \psi(x) = \varepsilon g(x) \qquad \text{for} \quad x \in \mathbb{R}^n. \tag{1.4}$$

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