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ON THE ASYMPTOTIC BEHAVIOR OF MINIMIZERS OF THE GINZBURG-LANDAU MODEL IN 2 DIMENSIONS

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Dedicated to the memory of Peter Hess

Abstract. Minimizers u_ϵ of the Ginzburg-Landau energy E_ϵ defined in (1) below on an arbitrary domain $\Omega \subset\subset \mathbf{R}^2$ with smooth boundary and boundary data $g: \partial\Omega \to S^1$ as $\epsilon \to 0$ are shown to subconverge weakly in $H^{1,p}$ for $p < 2$ and locally in $H^{1,2}$ away from finitely many points $x_1, \dots, x_J$ to a smooth harmonic map $u: \Omega \setminus \{x_1, \dots, x_J\} \to S^1$. The proof is based on simple comparison arguments. The result simplifies and extends previous work of Bethuel-Brezis-Hélein for the same problem on a star-shaped domain.

1. Introduction. Let Ω be a bounded domain in $\mathbf{R}^2$ with smooth boundary $\partial\Omega = \Gamma_1 \cup \dots \cup \Gamma_K$, where $\Gamma_k \cong S^1$ for $1 \le k \le K$, and let $g = (g_1, \dots, g_K)$ be smooth functions $g_k: \Gamma_k \to S^1 \subset \mathbf{C} \cong \mathbf{R}^2$, $1 \le k \le K$. Through the identification $\Gamma_k \cong S^1$ we may associate with each g_k a topological degree d_k.

Also let

$$H_g^1(\Omega) = \left\{u \in H^{1,2}(\Omega; \mathbf{R}^2) : u_{|\Gamma_k} = g_k,\ 1 \le k \le K\right\}.$$

It is well-known that $H_g^1(\Omega)$ is non-void. Moreover, for $\epsilon > 0$, $u \in H_g^1(\Omega)$ we define the Ginzburg-Landau energy

$$E_\epsilon(u) = E_\epsilon(u; \Omega) = \frac{1}{2}\int_\Omega \left\{|\nabla u|^2 + \frac{1}{2\epsilon^2}\left(1 - |u|^2\right)^2\right\} dx. \tag{1}$$

It is easy to see that for each $\epsilon > 0$ the infimum

$$\nu(\epsilon) = \inf_{u \in H_g^1} E_\epsilon(u)$$

is attained at a minimizer $u_\epsilon \in H_g^1$.

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