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STABILIZATION OF VISCOELASTIC FLUID MOTION (OLDROYD'S MATHEMATICAL MODEL)

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In memoriam Peter Hess

Abstract. The stabilization conditions of Oldroyd's viscoelastic fluid motion are found. The exact exponential estimates of stabilization rate are established.

1. The mathematical Oldroyd's model of viscoelastic fluid motion is investigated. Such model (see [1]) can be defined by reological relation

$$k_0\sigma + k_1 \partial\sigma/\partial t = \eta_0\xi + \eta_1 \partial\xi/\partial t, \quad k_1\sigma(0,x) = \eta_1\xi(0,x). \tag{1}$$

Here σ is the deviator of the stress tensor and ξ is the strain tensor. Namely, ξ is the $n \times n$ matrix with components $\xi ij = 1/2 \cdot (\partial v_i/\partial x_j + \partial v_j/\partial x_i)$, $v = (v_1, \dots, v_n)$ is a rate of the fluid motion and k_0, k_1, η_0, η_1 are positive constants, $n = 2, 3$. If $\eta_0 k_1 = k_0 \eta_1$ in (1), we shall obtain Newton's model of uncompressible viscous fluid.

Relation (1) and the motion equation in Cauchy form leads us to the initial boundary value problem

$$\begin{aligned}
&\frac{\partial v}{\partial t} + \sum_{k=1}^{n} v_k \cdot \partial v/\partial x_k - \varepsilon\Delta v - \int_0^t \rho \exp[-(t-s)\delta] \cdot \Delta v\, ds - \operatorname{grad} p = f,\\
&\operatorname{div} v = 0 \quad (t \geq 0, x \in \Omega); \quad v = 0 \ \ (t \geq 0, x \in \partial\Omega);\\
&v(0,x) = v^0(x) \ \ (x \in \bar{\Omega}); \quad (p,1) \equiv \int_\Omega p(t,x)dx = 0.
\end{aligned} \tag{2}$$

Here $\varepsilon = \eta_1/k_1$, $\rho = \eta_0/k_1 - (k_0\eta_1)/k_1^2$, $\delta = k_0/k_1$; Ω is an open bounded domain of points $x = (x_1, \dots, x_n)$ in R^n with boundary $\partial\Omega$ and $\bar{\Omega} = \Omega \cup \partial\Omega$. The last condition in (2) is introduced for uniqueness of the pressure p.

Problem (2) is the generalization of the first initial-boundary value problem for Navier-Stokes system.

For the first time, problem (2) was investigated in the articles of A.P. Oskolkov and his pupils (see [2]). They applied Ladyzenskaja's methods (see [3]). These investigations were continued in the article of Yu. Ya. Agranovich, P.E. Sobolevskii [4]. The existence and uniqueness of problem (2) solution, local in time for $n = 3$

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