

# NONLINEAR EIGENVALUES AND GLOBAL BIFURCATION APPLICATION TO THE SEARCH OF POSITIVE SOLUTIONS FOR GENERAL LOTKA-VOLTERRA REACTION DIFFUSION SYSTEMS WITH TWO SPECIES

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Dedicated to the memory of Peter Hess

**Abstract.** In this work we use an optimal version of the global bifurcation theorem by Rabinowitz to show the existence of global continua of component-wise positive solution couples for general systems of Lotka-Volterra type involving two species. No restriction on the number of semitrivial coexistence states is imposed.

**1. Introduction.** In this work we analyze the existence of component-wise positive solutions of general nonlinear boundary value problems of the form

$$\begin{aligned} L_1(D)u &= [\lambda \ell(x) + f(x, u) + F(x, u, v)u], \quad x \in \Omega, \\ L_2(D)v &= [\mu m(x) + g(x, v) + G(x, u, v)v], \quad x \in \Omega, \\ u|_{\partial\Omega} &= v|_{\partial\Omega} = 0, \end{aligned} \tag{1.1}$$

where we assume

$H_1$ .  $\Omega$  is a bounded domain of  $\mathbb{R}^d$ ,  $d \geq 1$ , with boundary  $\partial\Omega$  of class  $C^{2+\nu}$  for some  $\nu \in (0, 1)$ .

$H_2$ .  $L_k(D)$ ,  $k = 1, 2$ , are uniformly elliptic operators in  $\Omega$

$$L_k(D) = - \sum_{i,j=1}^d \alpha_{ij}^k(x) D_i D_j + \sum_{i=1}^d \alpha_i^k(x) D_i + \alpha_0^k(x), \quad k = 1, 2, \tag{1.2}$$

with all their coefficients  $\alpha_{ij}^k, \alpha_h^k$ , in the Banach space of Hölder continuous functions

$$Y := C^\nu(\overline{\Omega}; \mathbb{R}).$$

$H_3$ .  $\ell, m \in Y$  and  $\lambda, \mu \in \mathbb{R}$ . Moreover,  $\ell(x) > 0$  and  $m(x) > 0$  for all  $x \in \Omega$ .

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Received September 1993.

AMS Subject Classifications: 35J55, 35P30, 35B32.