Differential and Integral Equations, Volume 7, Number 5, September 1994, pp. 1367 – 1388.

OPTIMAL CONTROL PROBLEMS FOR NONLINEAR PARABOLIC BOUNDARY CONTROL SYSTEMS: THE DIRICHLET BOUNDARY CONDITION

H.O. FATTORINI*
University of California, Department of Mathematics, Los Angeles, California 90024

T. MURPHY
TRW Inc., Redondo Beach, California 90278

Dedicated to the memory of Peter Hess

Abstract. We consider optimal problems for boundary control systems, control acting through the Dirichlet boundary condition; constraints on the control and target conditions are included. The final result is a version of Pontryagin's maximum principle. Existence theorems are also obtained.

1. Introduction. Let Ω a bounded domain with boundary $\partial\Omega$ in m-dimensional Euclidean space $\mathbb{R}^m$. We consider a boundary control system described by a semilinear heat equation in Ω,

$$y_t(t,x) = \Delta y(t,x) + f(t, y(t,x)), \qquad (x \in \Omega, 0 < t \le T), \tag{1.1}$$

$$y(0,x) = \zeta(x), \qquad (x \in \Omega), \tag{1.2}$$

control applied through the Dirichlet boundary condition

$$y(t,x) = u(t,x), \qquad (x \in \partial\Omega,\ 0 < t \le T). \tag{1.3}$$

Controls are taken in $L^\infty([0,T] \times \partial\Omega)$. The optimal control problem is that of minimizing a **cost functional** $y_0(t,u)$ among all controls satisfying a constraint

$$u(t,\cdot) \in U = \text{ control set } \subseteq L^\infty(\partial\Omega) \ \ (0 \le t \le \bar{t}) \tag{1.4}$$

whose corresponding solutions $y(t,x,u)$ satisfy a **target condition**

$$y(\bar{t},\cdot,u) \in Y = \text{ target set}\,. \tag{1.5}$$

The time $\bar{t}$ at which the optimal process terminates may be fixed or free.

Received August 1993.
*Supported by the National Science Foundation under grant DMS-9221819.
AMS Subject Classifications: 93E20, 93E25.