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TURING INSTABILITIES FOR SYSTEMS OF TWO EQUATIONS WITH PERIODIC COEFFICIENTS

E.N. Dancer
School of Mathematics and Statistics, University of Sydney, Sydney, NSW 2006, Australia

To the memory of Peter Hess

Abstract. In this paper, we discuss Turing instabilities for systems of 2 reaction diffusion equations of population type and time periodic coefficients. We find interesting differences from the autonomous case. Our techniques also give some new examples of non–uniqueness for predator–prey models.

In this paper, we consider the problem

$$\begin{aligned}
\frac{\partial u}{\partial t} &= r_1(t)\Delta u + u(a(t) - b(t)u + c(t)v) \quad \text{in } \Omega \times [0, \infty) \\
\frac{\partial v}{\partial t} &= r_2(t)\Delta v + v(d(t) + e(t)u - f(t)v) \qquad (1) \\
\frac{\partial u}{\partial n} &= \frac{\partial v}{\partial n} = 0 \quad \text{on } \partial\Omega \times [0, \infty).
\end{aligned}$$

Here Ω is a bounded open set in $\mathbb{R}^m$ with smooth boundary and a, b, c, d, e, f, r_1, r_2 are T periodic and $r_1(t)$, $r_2(t) > 0$ always. Note that it is natural to consider periodic coefficients because of seasonal variations. Clearly, we can obtain some solutions of the partial differential equation by simply looking at solutions independent of $x \in \Omega$. Note that these clearly satisfy the boundary conditions. Thus we obtain some solutions by solving

$$\begin{aligned}
\frac{\partial u}{\partial t} &= u(a(t) - b(t)u + c(t)v) \\
\frac{\partial v}{\partial t} &= v(d(t) + e(t)u - f(t)v).
\end{aligned} \qquad (2)$$

Suppose that (u_0, v_0) is a strictly positive T periodic solution of (2) which is stable (as a solution of (2)). By strictly positive, we mean that both components are always strictly positive. We say that a Turing instability occurs if (u_0, v_0) is unstable as a solution of the partial differential equation (1). In the autonomous case and with u_0 and v_0 independent of time, this problem has been studied extensively (cp. Conway

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