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A KREIN SPACE APPROACH TO ELLIPTIC EIGENVALUE PROBLEMS WITH INDEFINITE WEIGHTS

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Dedicated to the memory of Peter Hess

1. Introduction. We consider the eigenvalue problem

$$Lu = \lambda r u, \tag{1}$$

where L is a symmetric elliptic operator and r is a locally integrable function on $\mathbb{R}^n$. If r is of constant sign then this problem leads to a selfadjoint problem in the Hilbert space $L^2(|r|)$. In this note we are interested in the case when r takes both positive and negative values on sets of positive measure. Then the spectrum of the problem (1) is not necessarily real any more. Moreover it is not apparent that the spectrum does not cover the whole complex plane. Even when it is discrete there can be nonsimple eigenvalues. For ordinary differential equations the related completeness problem of the eigenfunctions has been studied extensively in recent years; see for example [3, 6] and the references quoted therein. The corresponding question in the case of continuous spectrum has been addressed in [6].

When $n > 1$ some special properties of the spectrum of (1) have been considered in [1, 8-12]. In all of these papers the spectrum is discrete. To ensure the discreteness of the spectrum the problem (1) had to be considered on a bounded domain Ω with appropriate boundary conditions.

The important question of completeness of the eigenfunctions in the space $L^2(\Omega, |r|)$ has been recently considered in [19]. In that paper an operator-theoretic approach has been used to give sufficient conditions for the eigenvectors of the generalized selfadjoint eigenvalue problem $Su = \lambda Tu$ to form a Riesz basis. Applied to the problem (1) on the bounded domain Ω with the weight function r bounded and bounded away from zero and satisfying certain smoothness conditions, these results imply that eigenfunctions of (1) form a Riesz basis in $L^2(\Omega, |r|)$. In case that r is

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