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NONTRIVIAL SOLUTIONS FOR A STRONGLY RESONANT PROBLEM[1]

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Introduction. In this paper we will consider some applications of critical point theorems to the problem of existence of nontrivial solutions for a class of strongly resonant problems at a higher eigenvalue of $-\Delta$ with Dirichlet boundary condition on a bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 1$. More precisely, we will be concerned with the problem

$$-\Delta u = \lambda_k u + g(u) \quad \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega, \tag{P}$$

where λ_k, $k \geq 2$ is an eigenvalue of $-\Delta$ on $H_0^1(\Omega)$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function with subcritical growth satisfying the following Lipschitz bound from below

$$\frac{g(s) - g(t)}{s - t} \geq -\delta \quad \forall\ s, t \in \mathbb{R},\ s \neq t, \tag{g_0}$$

for some $0 < \delta < \lambda_k - \lambda_{k-1}$. Here we are denoting by $0 < \lambda_1 < \lambda_2 < \ldots < \lambda_j < \ldots$ the distinct eigenvalues of $-\Delta$ on $H_0^1(\Omega)$ and will let N_j denote the corresponding λ_j-eigenspace.

Since the appearance of the papers [28, 29] by Thews and [7] by Bartolo-Benci-Fortunato, several authors have studied the "strongly resonant" case, where one has

$$\lim_{s\to\pm\infty} g(s) = 0, \quad \lim_{s\to\pm\infty} G(s) = \lim_{s\to\pm\infty} \int_0^s g(\sigma)d\sigma = \beta \in \mathbb{R}. \tag{$(g_1)_\beta$}$$

We refer the reader to e.g. [2, 4-7, 9, 12-15, 17, 18] and the references therein. More specifically, the authors in [7] proved several existence and multiplicity results (in the presence of symmetries) for the strongly resonant case through the use of variational techniques under a weak compactness condition of the Palais-Smale type which had been previously introduced by Cerami in [11]. The main idea was to show that general abstract results of the minimax type (as in [24]) held true under the above mentioned weak compactness condition.

On the other hand, in [5] and [13] the authors considered other situations of strong resonance at the first eigenvalue λ_1, using standard variational methods under the usual Palais-Smale condition. Their approach was conceptually simpler and based on analyzing the levels $c \in \mathbb{R}$ where the localized Palais-Smale condition $(PS)_c$ held true.

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