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BIFURCATION FROM EIGENCURVES OF THE p-LAPLACIAN

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1. Introduction. We study the perturbed two parameter nonlinear eigenvalue problem

$$\begin{aligned} -\Delta_p u + f(x,u) - \lambda m(x)|u|^{p-2}u = \mu|u|^{p-2}u \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{1.1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $p \in (1,\infty)$, $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $m \in L^\infty(\Omega)$ may change sign, and $f(x,u)$ is a higher order perturbation term to be specified later. This is a development of the investigation in [11], where we concentrated on the existence of positive solutions and the asymptotic behavior of the associated eigencurve of (1.1) with $f \equiv 0$. We are mainly concerned here with the bifurcation properties of (1.1), and loosely speaking, we shall deal with open problem (3) of [11].

Starting with Richardson's work [23] on eigencurves for Sturm-Liouville problems, the two parameter eigenvalue problem of the form

$$Tu - \lambda Su = \mu Eu \tag{1.2}$$

has a long history, and has received growing attention recently, in particular in the case where T is a linear (often selfadjoint) ordinary or partial differential operator, S is a multiplication operator induced by a weight function, definite or not, and E is an identity in appropriate space. We mention in this aspect the works of Allegretto [1], Binding and Browne [7, 8], Binding, Browne, Huang and Picard [9], in which the focus is on the asymptotic behavior of the eigencurves. Replacing T by the p-Laplacian, which is a nonlinear operator, and modifying S and E correspondingly, Binding and Huang [11] studied the asymptotic behavior of the first eigencurve $\mu_1(\lambda)$. For general p, we obtained simplicity and continuity of the first eigencurve of (1.1), as well as positivity of the corresponding eigenfunction. See [11] for more motivation and more references. With the perturbation term $f(x,u)$ present, bifurcation from eigencurves has been treated by various authors. Alexander and Antman [2] studied bifurcation structure in a rather general framework. Using the implicit function theorem, Binding [6] considered bifurcation problems for abstract linear operators in Banach space, and Cantrell [13] treated Sturm-Liouville

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