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SPATIAL PATTERNS DESCRIBED BY THE EXTENDED FISHER–KOLMOGOROV (EFK) EQUATION: KINKS

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1. Introduction. In the study of spatial patterns, bistable systems play an important role. A typical example, which has been extensively studied in the context of population dynamics [1, 6, 9], leads to the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (u-a)(1-u^2), \qquad -1 < a < 1, \tag{1.1}$$

which is sometimes referred to as the Fisher-Kolmogorov or FK equation, or the Nagumo equation. In this context equation (1.1) describes the interaction between dispersal, modeled by the diffusion term, and survival fitness, represented by the function

$$f(u) = (u-a)(1-u^2), \qquad -1 < a < 1.$$

The two stable uniform states $u = \pm 1$ are separated by the unstable state $u = a$. If $a = 0$ equation (1.1) admits a stationary monotone transition layer solution connecting $u = -1$ and $u = +1$, which is unique, except for translations and given by

$$u(x) = \tanh\left(\frac{x}{\sqrt{2}}\right).$$

If $a \neq 0$ then (1.1) admits monotone travelling wave solutions connecting $u = -1$ and $u = +1$.

Recently, interest has turned to a higher order extension of the FK-equation – the EFK equation – of the form

$$\frac{\partial u}{\partial t} = -\gamma \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial x^2} + (u-a)(1-u^2), \quad -1 < a < 1,\ \gamma > 0. \tag{1.2}$$

The motivation for studying this equation has come in part from the fact that in phase transitions it describes the dynamics near a critical point where the coefficient of $|\nabla u|^2$ in the Landau free energy functional vanishes and the lowest order spatial derivatives that appear are of second order (a *Lifshitz point*) [8, 12]. However, like

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