Differential and Integral Equations, Volume 8, Number 7, September 1995, pp. 1619 – 1656.

Γ–CONVERGENCE, MINIMIZING MOVEMENTS AND GENERALIZED MEAN CURVATURE EVOLUTION

Irene Fonseca
Department of Mathematics, Carnegie Mellon University, Pittsburgh, PA 15213

Markos A. Katsoulakis
Department of Mathematics, Michigan State University, East Lansing, MI 48824

(Submitted by: D. Kinderlehrer)

1. Introduction. In this paper we show that *viscosity solutions* to *curvature evolution* equations may be obtained as limits of minimizers for Γ-limits of inhomogeneous, anisotropic singular perturbations for certain nonconvex variational problems. We consider the energy

$$I(u) := \int_\Omega W(u(x))\,dx,$$

where Ω is an open, bounded, strongly Lipschitz domain of $\mathbb{R}^N$, $u : \Omega \to \mathbb{R}^n$, and W supports two phases; i.e., W has two isolated (global) minimum points a and b. The minimization of the energy $E(\cdot)$ subject to fixed volume fraction θ, $0 < \theta < 1$, admits infinitely many solutions, which are piecewise constant measurable functions of the form $u = \chi_A a + (1 - \chi_A)b$, with $\mathrm{meas}(A) = \theta\ \mathrm{meas}(\Omega)$. In order to find a selection criterion for resolving this nonuniqueness, we fix an initial phase-a configuration, A_0, and we introduce the family of perturbed problems

$$I^h_\epsilon(u) := \int_\Omega W(u(x))\,dx + \int_\Omega \epsilon^2\Lambda^2(x, \nabla u(x))\,dx + \int_\Omega \epsilon f_h(x, u(x); A_0)\,dx,$$

where $\epsilon, h > 0$, $\partial^* A_0$ is the reduced boundary of A_0 (see Section 2) and $\Lambda(x, \cdot)$ has linear growth. A particularly interesting example of the last contribution to the total energy is given by the density

$$f_h(x, u; A_0) := |u - \chi_{A_0}(x)a - (1 - \chi_{A_0}(x))b|^p g\Big(\frac{d(x, \partial^* A_0)}{h}\Big), \tag{1.1}$$

where $d(x, \partial^* A_0)$ denotes the signed distance from x to $\partial^* A_0$.

In order to study the behavior of minimizing sequences, we rescale the energy to obtain

$$E^h_\epsilon(u; A_0) := \int_\Omega \frac{1}{\epsilon} W(u(x))\,dx + \epsilon \int_\Omega \Lambda^2(x, \nabla u(x))\,dx + \int_\Omega f_h(x, u(x); A_0)\,dx.$$

Received for publication November 1994.
AMS Subject Classifications: 35A05, 35K55, 49J45, 49L25, 49Q20, 53A10.