Differential and Integral Equations, Volume 8, Number 7, September 1995, pp. 1585 – 1590.

UNIFORM LIPSCHITZ REGULARITY OF A SINGULAR PERTURBATION PROBLEM

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The following free boundary problem has appeared in several contexts, in particular in the theory of flame propagation (see, for instance [2]): Study the uniform properties (and the limit) of solutions $u_\epsilon(x,t)$ in $\mathbb{R}^n \times (0,T)$ of the equation

$$\triangle u - u_t = \beta_\epsilon(u),$$

where β_ϵ is an approximation to Dirac's δ; i.e., β_ϵ

$$\text{support } \beta_\epsilon(u) = [0,\epsilon],$$

and

$$\int_0^\epsilon \beta_\epsilon(s)ds = 1.$$

The purpose of this work is to show that such solutions are locally uniformly Lipschitz in space (see [2] for estimates depending on the initial data).

In a recent paper in honor of Professor Magenes ([1]), we presented a monotonicity formula for caloric functions (solutions of the heat equation) in adjacent domains and sketched an application to a free boundary problem.

In this note we would also give complete details of this application in a somewhat more general context (Corollary 2).

The main result in this paper concerns solutions of a second-order parabolic equation for which homogeneous solutions have Lipschitz a priori estimates.

For simplicity we will consider the uniformly elliptic operator

$$Lw = D_t w - D_i(F_i(\nabla w))$$

or

$$Lw = D_t w - F(D^2 w)$$

and solutions of the singular perturbation problem

$$|Lu| \le \frac{1}{\epsilon}\chi_{0<u<\epsilon}.$$

Received for publication January 1995.
AMS Subject Classifications: 35J60.