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THE EXISTENCE OF SIMILARITY SOLUTIONS TO A QUASILINEAR PARABOLIC EQUATION

Yuan-wei Qi
Department of Mathematics, HKUST, Hong Kong

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1. Introduction. In this paper we study the existence of non-constant positive solution of the following differential equation

$$\triangle w + w^p - (\frac{1}{2} y \cdot \bigtriangledown w + \lambda w) w^{-q} = 0, \qquad y \in \mathbb{R}^n, \tag{1}$$

where $0 < q < 1$, $\lambda > 0$ and $p > 1$. The main interest of (1) is its relation with blow up self-similar solution of the porous media equation

$$\phi_t = \bigtriangledown \cdot (\phi^\sigma \bigtriangledown \phi) + \phi^\beta, \tag{2}$$

where $\sigma > 0$ and $\beta > \sigma + 1$. Indeed, when $\lambda = 1/(p-1)$, (1) is the equation for blow up self-similar solution of (2) with $q = \sigma/(\sigma+1)$ and $p = \beta/(\sigma+1)$, as was shown in [14]. For simplicity, we shall consider the radial symmetric case which results in

$$w'' + \frac{n-1}{r} w' + w^p - (\frac{r}{2} w' + \lambda w) w^{-q} = 0, \quad w'(0) = 0, \quad w(0) = \eta > 0. \tag{3}$$

This paper is a continuation of [14] on the study of (1). In fact, the existence of positive solution of (3) for $\lambda = 1/(p-1)$, $1 < p < p_c$ was proved and asymptotic behavior of positive solution is established in [14]. Here, the number p_c is defined to be

$$p_c = \begin{cases} \infty, & n \le 2 \\ \frac{n+2}{n-2}, & n > 2. \end{cases} \tag{4}$$

The purpose of this paper is (i) to use the techniques of [14] and a Pohožaev type identity to find out for what value of λ (3) has a non-constant positive solution when $1 < p < p_c$ and (ii) to approach the case $\lambda = 1/(p-1)$ when $p > p_c$ by employing some new techniques.

Theorem 1. *Let* $1 < p < p_c$.

(a) *There exists a non-constant positive solution of* (3) *when* $\lambda(p+q-1) > 1$.
(b) *There exists no non-constant positive solution of* (3) *when* $\lambda(p+q-1) \le 1$.

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