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REGULARIZATION OF AN INVERSE STEFAN PROBLEM

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1. Introduction. The melting of a thin block of ice occupying an interval $b \le x < \infty$ and being at temperature $0^\circ C$ everywhere in this interval is described by the one-dimensional Stefan problem

$$\partial^2 u/\partial x^2 - \partial u/\partial t = 0 \quad \text{in } 0 < x < s(t), \quad 0 < t, \tag{1.1}$$

along with boundary conditions

$$u(s(t), s) = 0, \quad t > 0, \tag{1.2}$$
$$(\partial u/\partial x)(s(t), t) = -\mu \cdot (ds/dt)(t), \quad t > 0, \tag{1.3}$$
$$(\partial u/\partial x)(0, t) = v(t), \quad t > 0 \tag{1.4}$$

and initial data

$$s(0) = b, \quad u(x, 0) = u_0(x), \quad 0 < x < b \tag{1.5}$$

in which u represents the temperature of the liquid phase and s the position of the melting interface.

Given the initial distribution u_0, the so-called Inverse Stefan problem is to find, to a prescribed interface s, a time dependent heat flux v such that the problems (1.1)–(1.5) have a continuous solution on $0 \le x \le s(t)$, $0 \le t$. As is known, this is an ill-posed problem, a particular case of which is the so-called Cauchy problem for the heat equation, corresponding to $s(t) = a$ constant. The Cauchy problem for the heat equation has been the object of extensive literature (cf. the bibliography at the end). Note that an ill-posed problem, as such, is intractable, numerically. In order for numerical computations to be possible, one has to regularize it, i.e., approximate it by a well-posed problem. To determine the degree of accuracy, in other words, to derive error estimates, is of vital importance, both theoretically

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