Differential and Integral Equations Volume 10, Number 3, May 1997, pp. 437 – 454.

DIFFUSION EQUATION WITH A GENERAL POLYNOMIAL PERTURBATION

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(Submitted by: S. Klainerman)

Introduction. Nonlinear perturbations of the heat equation have been considered by many authors. In [7] Klainerman proved the global existence, uniqueness and certain decay properties for solutions of a large class of nonlinear heat equations with small initial data. The complicated Nash-Moser-Hörmander iteration method of [7] has later been replaced by more simple methods; see the book [9]. Recently, in the work [1] very accurate information on the asymptotic behavior of the nonlinear heat equation was obtained using the renormalization group (RG) method.

In the above-mentioned works the nonlinear perturbation is a function of the unknown function u and its derivatives with respect to x. (See (0.1) for notation.) In the present work we consider a larger class of nonlinear perturbations, including even integral operators. In Theorem 3.3 we describe the asymptotic behavior (as $t \to \infty$) of the solution to the Cauchy problem with small initial data f

$$\frac{\partial u}{\partial t} = \Delta u + F_p(u) \ , \quad t \in [1, \infty), \quad x \in \mathbb{R}^N \tag{0.1}$$

$$u(x, 1) = f(x) \ , \qquad x \in \mathbb{R}^N, \tag{0.2}$$

where $u : [1, \infty) \times \mathbb{R}^N \to \mathbb{C}$ is unknown, $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$, $\Delta = \sum_{j=1}^N \frac{\partial^2}{\partial x_j^2}$ and F_p belongs to a quite general class of polynomial operators. We shall show that the solution of (0.1)–(0.2) behaves asymptotically as the solution of the unperturbed equation. The class of perturbations contains certain polynomial PDO's with variable coefficients and also many nonlinear integral operators, like

$$F_p(f)(x) = \int_{\mathbb{R}^N} K(x - y) \prod_{j=1}^m D^{n_j} f(y) \, dy,$$

where $m \geq 2$, $n_j \in \mathbb{N}^N$, $0 \leq |n_j| \leq 2$, $\sum_{j=1}^m |n_j| > 2 - (m-1)N$ and the kernel $K : \mathbb{R}^N \to \mathbb{R}^N$ is assumed to satisfy $K(x) \in L^1(\mathbb{R}^N)$ and $x_\iota K(x) \in L^1(\mathbb{R}^N)$ for every $\iota = 1, \ldots, N$; see Proposition 4.3. See also the general definition of F_p in Section 1, 5°, and the other examples in Section 4.

Received for publication February 1996.
AMS Subject Classifications: 35K55, 35B40.