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ON A FREE-BOUNDARY PROBLEM FOR BURGERS EQUATION: THE LARGE-TIME BEHAVIOUR

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1. Introduction. In this paper we describe the large-time behavior of the solution $(u(x,t),\zeta(t))$ of the free-boundary problem

$$\text{(FB)}\qquad \begin{cases} u_t = u_{xx} + uu_x & x \in \mathbf{R}\setminus\{\zeta(t)\},\ t>0 \\ u(\zeta(t)^-,t) = u(\zeta(t)^+,t) = q & t>0 \\ u_x(\zeta(t)^-,t) - u_x(\zeta(t)^+,t) = 1 & t>0 \\ u(x,0) = u_0(x) & x \in \mathbf{R} \\ \zeta(0) = \zeta_0, \end{cases}$$

where q is a positive constant, ζ_0 a given real number and u_0 is a given initial function satisfying the hypothesis

H. $u_0 \in C(\mathbf{R}) \cap \{C^3((-\infty,\zeta_0]) \cup C^3([\zeta_0,\infty))\}$, $0 \le u_0 < q$ in $(-\infty,\zeta_0)$, $q < u_0 \le A$ in (ζ_0,∞) for some $A > q$, $u_0'(\zeta_0^-) - u_0'(\zeta_0^+) = 1$, $u_0'(x) \to 0$ as $x \to \pm\infty$, $u_0'(\zeta_0^+) > 0$, and $u_0 - AH \in L^1(\mathbf{R})$, where H denotes the Heaviside function.

Problem (FB) arises in combustion theory. For a brief account of the physical background of the problem we refer to [3].

The well-posedness of Problem (FB) has been proved by Bertsch, Hilhorst and Schmidt-Lainé ([1]). Their main observation was that, if u_0 satisfies H, Problem (FB) is formally equivalent to the problem

$$\text{(P)}\qquad \begin{cases} u_t = u_{xx} + uu_x + (H(u-q))_x & x \in \mathbf{R}, t>0 \\ u(x,0) = u_0(x) & x \in \mathbf{R}, \end{cases}$$

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