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A REMARK ON THE CONTINUOUS DEPENDENCE ON ϕ OF SOLUTIONS TO $U_T - \Delta\phi(U) = 0$

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1. Introduction. In this note we examine the singular Cauchy problem

$$\begin{cases} u_t - \Delta \ln u = 0 & \text{in } \mathbb{R}^N \times (0,T) \\ u(x,0) = u_0(x) \geq 0. \end{cases} \tag{1}$$

The partial differential equation in (1) arises in the dynamics of thin liquid films (see [8] and references therein), the expansion of an electron cloud (see [16]), and the Ricci flow on $\mathbb{R}^2$ (see [19]). It also arises formally as the limit as $m \to 0$ of the porous medium equation.

Through a rescaling in time the porous medium equation can be written

$$u_t - \frac{1}{m}\Delta u^m = 0 \quad \text{for all } 0 < m < 1$$

or equivalently

$$u_t - \Delta\Big(\frac{u^m - 1}{m}\Big) = 0.$$

Of course, for $u > 0$

$$\frac{u^m - 1}{m} \to \ln u \quad \text{as} \quad m \to 0.$$

Thus (1) can be viewed, at least formally, as the limit as $m \to 0$ of

$$\begin{cases} u_t - \frac{1}{m}\Delta u^m = 0 & \text{in } \mathbb{R}^N \times (0,T) \\ u(x,0) = u_0(x) \geq 0. \end{cases} \tag{2}$$

The partial differential equation in (2) arises in, for example, thermoconduction (see [15], Chapter 5), plasma physics (see [3]), and modeling the diffusion of impurities in silicon (see [12]).

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