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# ON THE BASIC CONCENTRATION ESTIMATE FOR THE GINZBURG-LANDAU EQUATION

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**1. Introduction.** Let $\Omega$ be a bounded, smooth domain in $\mathbb{R}^2$. We consider the Ginzburg-Landau system,

$$\begin{aligned} -\Delta u &= \frac{1}{\varepsilon^2}(1-\mid u\mid^2)u \quad \text{in } \Omega, \\ u &= g \quad \text{on } \partial\Omega, \end{aligned} \tag{1.1}$$

where $u:\bar{\Omega}\to\mathbb{R}^2$, $g:\partial\Omega\to S^1$ is smooth and $\varepsilon>0$ is a parameter. Associated to problem (1.1) is the energy functional

$$E_\varepsilon(u)=\frac{1}{2}\int_\Omega \mid\nabla u\mid^2+\frac{1}{4\varepsilon^2}\int_\Omega(1-\mid u\mid^2)^2, \tag{1.2}$$

whose critical points in $H_g^1=\{u\in H^1(\Omega,\mathbb{R}^2): u|_{\partial\Omega}=g\}$ correspond precisely to solutions of problem (1.1). The asymptotic behavior of these solutions as $\varepsilon\to 0$ has attracted considerable interest in recent years. Bethuel, Brezis and Hélein in [1] have described in detail the limit as $\varepsilon\to 0$ of the global minimizers of (1.2) in the case $\Omega$ starshaped.

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