Differential and Integral Equations Volume 12, Number 2, March 1999, 167–182.

LEFT DEFINITE STURM-LIOUVILLE PROBLEMS WITH EIGENPARAMETER DEPENDENT BOUNDARY CONDITIONS*

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1. Introduction. In this paper we continue our study of the second order linear equation

$$-(py')' + qy = \lambda ry \quad \text{on} \quad [0,1], \text{ where } p > 0, \quad ' = \frac{d}{dx}\ , \tag{1.1}$$

subject to boundary conditions of the form

$$(a_j\lambda + b_j)y(j) = (c_j\lambda + d_j)(py')(j), \tag{1.2}$$

where $(a_j, b_j, c_j,\ d_j) \neq \mathbf{0}$, $j = 0, 1$. Cases analysed previously [3, 4] required $r > 0$: a typical 'right definiteness' condition. We refer to [7, 12] for many references to the topic. Here we shall forego this assumption but replace it by 'left definiteness' conditions described in detail in Section 2. The first case of this kind in the literature seems to be Bôcher's analysis of the left definite problem (in particular, $q \geq 0$) with λ-independent boundary conditions, which he converted to a λ-dependent problem (cf. [9, p. 236]). Since then Ibrahim and Sleeman [8] have analysed eigenfunction completeness for the case $q \equiv 0$, $a_1 = d_1 = 1$, $b_1 = -1$, $c_1 = 0$ and a (λ-independent) Dirichlet

Received for publication May 1997
*Research supported in part by grants from the NSERC of Canada.
AMS Subject Classifications: 34B24, 34L20, 47A75