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A SHARPER DECAY ESTIMATE FOR THE QUASILINEAR WAVE EQUATION WITH VISCOSITY IN TWO SPACE DIMENSIONS

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1. Introduction. In this paper we are concerned with a decay property of solutions of the quasilinear wave equation with a strong dissipation:

$$u_{tt} - \operatorname{div}\{\sigma(|\nabla u|^2)\nabla u\} - \Delta u_t = 0 \quad \text{in } \Omega \times [0,\infty) \tag{1.1}$$

with the initial-boundary conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \text{ and } u|_{\partial\Omega} = 0, \tag{1.2}$$

where Ω is a bounded domain in R^2 with a C^2 class boundary $\partial\Omega$ and σ is a nonlinear function like $\sigma(v^2) = 1/\sqrt{1+v^2}$.

Let us consider the typical case $\sigma = 1/\sqrt{1+v^2}$. This equation was introduced by Greenberg [3] for the one dimensional case: $\Omega \subset R^1$, and the global existence and exponential decay of smooth solutions were proved by Greenberg [4], Greenberg, Mizel and MacCamy [5] and Yamada [13]. For N-dimensional case $\Omega \subset R^N$, the global existence and exponential decay of small amplitude solutions with small data were proved by Ebihara [2] and Kawashima and Shibata [7]. For large data in N-dimensional case, Kobayashi, Pecher and Shibata [8] proved the global existence of smooth solutions. In [8], however, no decay property of solutions is given for such solutions.

Recently in [10], the present author has proved that if the mean curvature $H(x)$ of $\partial\Omega$ at $x \in \partial\Omega$ is nonpositive, then for $(u_0, u_1) \in H_2 \cap H_1^0 \times H_1^0$, the

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