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REAL AND COMPLEX REGULARITY ARE EQUIVALENT FOR HYPERBOLIC CHARACTERISTIC VARIETIES

Guy Métivier
MAB, 351 Cours de la Libération, Talence 33405, France

Jeffrey Rauch[1]
Department of Mathematics, 525 East University Avenue, Ann Arbor MI 48109

(Submitted by: J.L. Bona)

If $P(\eta)$ is a real homogeneous polynomial one associates real and complex algebraic varieties

$$\mathbf{V_R} := \{\eta \in \mathbf{R}^n \setminus 0 \ : \ P(\eta) = 0\} \quad \text{and} \quad \mathbf{V_C} := \{\eta \in \mathbf{C}^n \setminus 0 \ : \ P(\eta) = 0\},$$

with $\mathbf{V_R} \subset \mathbf{V_C}$.

Definition. A homogeneous polynomial is **hyperbolic** with timelike direction $\theta \in \mathbf{R}^n \setminus 0$ if and only if for all real η the equation $P(\eta + s\theta) = 0$ has only real roots s (see [5], [4]).

In the trivial case of P being a constant, both varieties are empty. Taking $\eta = 0$ shows that $P(\theta) \neq 0$.

If P is of degree $m \geq 1$, then for each $\eta \in \mathbf{R}^n$, the equation $P(\eta + s\theta) = 0$ has m real roots counting multiplicity, so the line $\eta + s\theta$ cuts the varieties $\mathbf{V}$ in at least 1 and no more than m points. It follows that $\mathbf{V_R}$ (respectively $\mathbf{V_C}$) is a real algebraic variety (respectively algebraic variety) of real (respectively complex) codimension equal to one. $\mathbf{V_R}$ is called the **characteristic variety**.

The fundamental stratification theorems of real and complex algebraic geometry (see [1], [3]) imply that with the exception of a set of real or complex codimension 2, the varieties $\mathbf{V_R}$ and $\mathbf{V_C}$ are locally real analytic and analytic. That means on a neighborhood of a nonexceptional point $\underline{\eta}$ there is a real analytic function (respectively analytic function) $\phi(\eta)$ with $\phi(\underline{\eta}) = 0$ and $d\phi(\underline{\eta}) \neq 0$ whose zero set coincides with the variety. The nonexceptional points are called **regular** according to the next definition.

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