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ON A NONLINEAR VARIANT OF THE BEAM EQUATION WITH WENTZELL BOUNDARY CONDITIONS

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1. Introduction

A mathematical model for the transverse deflection of an extensible beam whose ends are held at fixed distance apart is

$$\frac{\partial^2 u}{\partial t^2}+\alpha\frac{\partial^4 u}{\partial x^4}+\Big(\rho+k\int_0^1\Big(\frac{\partial u}{\partial \xi}(\xi,t)\Big)^2 d\xi\Big)\Big(-\frac{\partial^2 u}{\partial x^2}\Big)=0 \tag{1.1}$$

$$u\,(0,t)=u\,(1,t)=\frac{\partial^2 u}{\partial x^2}\,(0,t)=\frac{\partial^2 u}{\partial x^2}\,(1,t)=0, \tag{1.2}$$

which has been proposed by Woinowsky and Krieger [20]. Here $k,\alpha>0$ are constants, $\rho\in\mathbf{R}$, and the nonlinear term represents the change in tension of the beam due to its extensibility. The model has also been discussed by Eisley [8], Dickey [7], and Ball [1]–[2], while related experimental results have been given by Burgreen [6].

Nonlinear beams have been the subject of much recent activity. Ball uses a Galerkin method to obtain weak solutions to (1.1) and obtains classical solutions by placing further restrictions on the regularity of the data. The abstract formulation of (1.1)–(1.2) is the equation

$$u_{tt}+\alpha A^2u+M\big(\big\|A^{\frac12}u\big\|^2\big)Au=0, \tag{1.3}$$

where A is a positive self-adjoint operator in a Hilbert space H and M is a real function. This model has been studied by Medeiros [16]. He supposed that $M\in C^1[0,\infty)$ be such that $M(\lambda)\geq m_0+m_1\lambda$, for any $\lambda; m_0,m_1>0$, and with A having compact resolvent. The same equation (1.3), but with a dissipative term, was studied by Brito [4]–[5], Pereira [17], and Holmes and Marsden [15].

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