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LOCAL WELL POSEDNESS OF THE CAUCHY PROBLEM FOR THE LANDAU-LIFSHITZ EQUATIONS

Atsushi Fuwa and Masayoshi Tsutsumi
Department of Applied Physics, School of Science and Engineering
Waseda University, Tokyo 169-8555, Japan

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1. Introduction

In 1935 Landau and Lifshitz derived an equation which describes the dynamics of the magnetization in continuum ferromagnetic bodies. The equation is of the form

$$\begin{cases} \dfrac{\partial u}{\partial t} = u \times \Delta u - \lambda u \times (u \times \Delta u), \\ u \,:\, \mathbb{R}^d \to \mathbb{S}^2, \end{cases} \tag{1.1}$$

where $\mathbb{S}^2$ denotes the two-dimensional sphere ([13]). The positive number λ is a damping parameter. (1.1) is usually called the Landau-Lifshitz equation. In case $\lambda = 0$, (1.1) is called the Heisenberg equation. This paper is devoted to studying the well posedness of the Landau-Lifshitz equation for $\lambda > 0$. Since it is a fully nonlinear system of parabolic partial differential equations and has a supercritical nonlinearity, many basic questions on the well posedness remain unsolved especially for higher spacial dimensional cases. We here assume that $\lambda = 1$.

The main previous results for the above equations are as follows. The first mathematical work on the Heisenberg equation is done by P. L. Sulem, C. Sulem, and C. Bardos [14] who establish the time local existence, uniqueness, and smoothness on $\mathbb{R}^d (d \geq 1)$. As to the Landau-Lifshitz equation Carbou and Fabrie [6] establish the time local existence, global existence with small initial data, uniqueness, and smoothness on $\mathbb{R}^3$. Zhou Yulin, Guo Boling, and Tan Shaobin prove global existence, uniqueness, and smoothness on $\mathbb{R}^1$, $\mathbb{T}^1 (\mathbb{T} = \mathbb{R}/\mathbb{Z})$ in [16]. Guo Boling and Min-Chun Hong establish global

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