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MINIMIZATION OF A GINZBURG-LANDAU TYPE ENERGY WITH POTENTIAL HAVING A ZERO OF INFINITE ORDER

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1. Introduction

Let G be a bounded and smooth, simply connected domain in $\mathbb{R}^2$ and let $g : \partial G \to S^1$ be a boundary condition of degree $\deg(g, \partial G) = d \geq 0$ (as we may assume without loss of generality). Consider a C^2 functional $J : \mathbb{R} \to [0, \infty)$ satisfying the following conditions:

(H_1) $J(0) = 0$ and $J(t) > 0$ on $(0, \infty)$,
(H_2) $J'(t) > 0$ on $(0, 1]$,
(H_3) there exists $\eta_0 > 0$ such that $J''(t) > 0$ on $(0, \eta_0)$.

For $\varepsilon > 0$ consider the energy functional

$$E_\varepsilon(u) = \int_G |\nabla u|^2 dx + \frac{1}{\varepsilon^2} \int_G J(1 - |u|^2)\, dx \tag{1.1}$$

over

$$H^1_g(G, \mathbb{C}) := \{u \in H^1(G, \mathbb{C}) : u = g \text{ on } \partial G\}. \tag{1.2}$$

It is easy to see that $\min_{u \in H^1_g(G,\mathbb{C})} E_\varepsilon(u)$ is achieved by some smooth u_ε which satisfies:

$$\begin{cases} -\Delta u_\varepsilon = \dfrac{1}{\varepsilon^2} j(1 - |u_\varepsilon|^2) u_\varepsilon & \text{in } G, \\ \quad u_\varepsilon = g & \text{on } \partial G, \end{cases} \tag{1.3}$$

where $j(t) := J'(t)$. The case $J(u) = (1 - |u|^2)^2$, corresponding to the Ginzburg-Landau (GL) energy, was studied by Bethuel, Brezis and Hélein [1, 2] (see also Struwe [5]), where it was shown that:

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