ERRATA TO "THE CAUCHY PROBLEM FOR THE SEMI-LINEAR QUINTIC SCHRÖDINGER EQUATION IN 1D", DIFFERENTIAL INTEGRAL EQUATIONS, 18 (2005), NO. 8, 947–960.

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In the paper [4] we considered the quintic nonlinear Schrödinger equation (NLS) in one dimension (1d)

$$\begin{aligned} &iu_t + u_{xx} - |u|^4 u = 0 \\ &u(x,0) = u_0(x) \in H^s(\mathbb{R}), t \in \mathbb{R}. \end{aligned} \tag{0.1}$$

The main result of the paper was that the initial-value problem (IVP) is globally well posed for initial data in $H^s(\mathbb{R})$ for any $s > \frac{4}{9}$. A crucial part of the proof is the validation of Proposition 1 in [4]. But this proposition is false as the following counterexample, pointed out to us by Vedran Sohinger, shows. More precisely consider the multiplier

$$M_6(\xi_1, \xi_2, ..., \xi_6) = \frac{m_1^2\xi_1^2 - m_2^2\xi_2^2 + m_3^2\xi_3^2 - m_4^2\xi_4^2 + m_5^2\xi_5^2 - m_6^2\xi_6^2}{\xi_1^2 - \xi_2^2 + \xi_3^2 - \xi_4^2 + \xi_5^2 - \xi_6^2}$$

defined on $\Gamma_6 = \{(\xi_1, \xi_2, \ldots, \xi_6) \in \mathbb{R}^6 : \xi_1 + \xi_2 + \ldots + \xi_6 = 0\}$. The reader can consult [4] for the formula that gives the multiplier $m_i,\ i = 1, 2, ..., 6$. Then if we choose $(\xi_1, \xi_2, \ldots, \xi_6) = K(5, -3, 6, -2, 1, -7)$ where K a very large number, we have $\xi_1 + \xi_2 + \ldots + \xi_6 = 0$ and $\xi_1^2 - \xi_2^2 + \xi_3^2 - \xi_4^2 + \xi_5^2 - \xi_6^2 = 0$. From the definition of m for very high frequencies, and for a given Sobolev regularity index $0 < s < 1$, we can see that M_6 fails to be bounded on Γ_6. Therefore the result of [4] is not correct. In a different paper, [1], the author in collaboration with D. De Silva, N. Pavlović, and G. Staffilani, used the bound of Proposition 1, to improve the global well posedness of the IVP for any $s > \frac{1}{3}$. As we show in [2], the method in [1] can be proved to be independent of the bound for M_6 and thus the result in [1] is correct as claimed. Therefore, the quintic nonlinear Schödingers equation on $\mathbb{R}$ is globally well posed for any $s > \frac{1}{3}$. Proposition 1 in [4] was also used in the

Accepted for publication: October 2009.