Differential and Integral Equations, Volume 8, Number 1, January 1995, pp. 213–222.

POSITIVE SOLUTIONS FOR THE ONE-DIMENSIONAL p-LAPLACIAN*

R. MANÁSEVICH
Departamento de Ingeniería Matemática
F.C.F.M. Universidad de Chile, Casilla 170, Correo 3, Santiago, Chile

F.I. NJOKU
Department of Mathematics, University of Nigeria, Nsukka, Nigeria

F. ZANOLIN
Dipartimento di Matematica e Informatica, Università, Via Zanon 6, 33100 Udine, Italy

(Submitted by: A.R. Aftabizadeh)

Introduction. In this work, we are concerned with the existence of positive solutions for the nonlinear two-point boundary value problem

$$(\phi_p(u'))' + f(t,u) = 0, \qquad u(a) = 0 = u(b), \tag{P_p}$$

where $' = \frac{d}{dt}$, $\phi_p(s) := |s|^{p-2}s$ and $(\phi_p(u'))'$ is the one-dimensional p-Laplacian, $p > 1$. Throughout the paper $\mathbb{R}_+ := [0, +\infty)$, and $f : [a,b] \times \mathbb{R}_+ \to \mathbb{R}$ is Caratheodory, that is, $f(\cdot, s)$ is measurable for all $s \in \mathbb{R}_+$, $f(t, \cdot)$ is continuous for almost every $t \in [a,b]$ and for every $r > 0$ there is an L^1-function η_r such that $|f(t,s)| \le \eta_r(t)$, for all $s \in [0,r]$ and almost every $t \in [a,b]$. We suppose also that $f(t,0) \equiv 0$.

By a solution to (P_p) we mean a function $u : [a,b] \to \mathbb{R}_+$, of class C^1, with $\phi_p(u')$ absolutely continuous, satisfying (P_p). A solution u is positive if $u(t) > 0$ for all $t \in (a,b)$.

In a recent paper, Kaper, Knapp and Kwong [9, Theorem 2], proved the existence of at least one positive solution to (P_p) by assuming f continuous and satisfying

$$(k_1) \qquad \lim_{u \to 0^+} \frac{f(t,u)}{\phi_p(u)} = l \le 0, \quad \text{and} \quad \lim_{u \to +\infty} \frac{f(t,u)}{\phi_p(u)} = +\infty,$$

where the limits are supposed to hold uniformly with respect to $t \in (a,b)$. Condition (k_1) is related to previous works concerning the existence of positive solutions for nonlinear elliptic problems where the differential operator is linear, that is, $p = 2$. Indeed, we recall from [6, Theorem 2.1] that for $f = f(u)$, the Dirichlet problem

$$\Delta u + f(u) = 0, \qquad u_{|\partial\Omega} = 0 \tag{D}$$

has at least one positive solution if

$$f(0) = 0 \quad \text{and} \quad f(u) \ge 0 \quad \text{for all} \quad u \in \mathbb{R}_+ \tag{1.1}$$

Received Feburary 1992, in revised form October 1993.

*This research was sponsored by FONDECYT-1209-91, by the EC for RM and FZ, and by MURST 60% (1990), Univ. of Udine for FZ.

AMS Subject Classification: 34B15.