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DETERMINATION OF AN UNKNOWN RADIATION TERM IN HEAT CONDUCTION PROBLEM

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1. Introduction. This paper seeks to determine an unknown radiation term $P(u)$ that depends only on the temperature at $x = 0, u(0,t)$, in a linear diffusion equation. Such problems arise in a variety of physical situations, for example in radiative heat transfer where the rate of radiation depends on the temperature $u(0,t)$. We shall identify both functions $u(x,t)$ and $P(u)$ from the following initial boundary value problem:

$$u_t = u_{xx}, \quad 0 < x < 1, \quad 0 < t < T, \tag{1.1}$$

with initial condition

$$u(x,0) = f(x), \quad 0 < x < 1, \tag{1.2}$$

and with boundary conditions

$$u_x(1,t) = g(t), \qquad 0 < t < T, \tag{1.3}$$

$$u_x(0,t) = P(u(0,t)) - h(t), \quad 0 < t < T. \tag{1.4}$$

If $P(u)$ were known, then (1.1)–(1.4) would provide a well-posed problem for $u(x,t)$. We need, therefore, an additional condition and we choose to impose the condition

$$u(0,t) = \psi(t), \quad 0 < t < T. \tag{1.5}$$

It is the purpose of this paper to show that there is a unique pair of solutions (P,u) to the inverse problem (1.1)–(1.5). The above problem was studied in the case $f(x) = 0$, $h(t) = 0$ setting in [1], and it was shown that there is a unique solution to this problem. The heat equation with non-linear conditions has been studied by several authors (cf. [2]-[7]).

In the next section of this paper, we will consider the problem (1.1)-(1.4) to provide a unique solution to this problem. In section 3 we will discuss the existence and uniqueness solution of the inverse problem (1.1)–(1.5). The final section describes some unicity and stability results.

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