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EXPANSION THEOREMS FOR A CLASS OF REGULAR INDEFINITE EIGENVALUE PROBLEMS

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Abstract. By using detailed asymptotic estimates for the Greens function of regular indefinite eigenvalue problems we prove theorems on the pointwise convergence of the corresponding eigenfunction expansions, generalizing the classical results on trigonometric Fourier series.

1. Introduction. In this article we consider boundary eigenvalue problems of the form

$$\ell(y) = y^{(n)} + \sum_{\nu=2}^{n} f_\nu(x) y^{(n-\nu)} = \lambda r(x) y, \quad x \in [0,1] \tag{1.1}$$

$$U_\nu(y) = U_{\nu 0}(y) + U_{\nu 1}(y) = 0, \quad 1 \le \nu \le n. \tag{1.2}$$

We assume that $r : [0,1] \to \mathbb{R}\backslash\{0\}$ is a step function, $f_\nu \in L[0,1]$, $2 \le \nu \le n$, and that the boundary conditions (1.2) are normalized; this means that

$$\begin{aligned} U_{\nu 0}(y) &= \alpha_\nu y^{(k_\nu)}(0) + \sum_{j=0}^{k_\nu - 1} \alpha_{\nu j} y^{(j)}(0), \\ U_{\nu 1}(y) &= \beta_\nu y^{(k_\nu)}(1) + \sum_{j=0}^{k_\nu - 1} \beta_{\nu j} y^{(j)}(1), \end{aligned} \tag{1.3}$$

where $\alpha_{\nu j}, \beta_{\nu j} \in \mathbb{C}$,

$$|\alpha_\nu| + |\beta_\nu| > 0 \quad \text{for} \quad 1 \le \nu \le n,$$

$$n-1 \ge k_1 \ge k_2 \ge \cdots \ge k_n \ge 0 \quad \text{with} \quad k_{\nu+2} < k_\nu \quad \text{for} \quad 1 \le \nu \le n-2,$$

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