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PERIODIC SOLUTIONS OF DISSIPATIVE DYNAMICAL SYSTEMS WITH SINGULAR POTENTIALS

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Abstract. We give sufficient conditions for the existence of solutions of the periodic boundary value problem

$$\ddot{u} + \frac{d}{dt}[\nabla f(u)] + \nabla g(u) = h(t), \qquad u(0) = u(T), \quad \dot{u}(0) = \dot{u}(T),$$

where $u \in \mathbb{R}^n$. The potential g is supposed to be singular at the origin and to satisfy a condition similar to the "strong force" assumption of W. Gordon. An extension is worked out for systems with several singularities.

1. Introduction. We consider the periodic boundary value problem

$$\ddot{u} + \frac{d}{dt}[\nabla f(u)] + \nabla g(u) = h(t), \qquad u(0) = u(T), \quad \dot{u}(0) = \dot{u}(T), \qquad (1.1)$$

under the general assumption that $g(u)$ becomes infinite as $u \to 0$ and look for solutions $u(t)$ of (1.1) that do not cross the singularity $u = 0$.

In the last two years there has appeared a rich literature on this problem in the absence of dissipation; i.e., when $f \equiv 0$. Such contributions go back to the work of Gordon [5] who introduced the "strong force" assumption which characterizes the behaviour of $g(u)$ near the singularity. In one way or another, this condition has been further explored, either for potentials of attractive type (see, e.g., [1], [2], [3], [6] and their references) or for potentials of repulsive type [4]. The method used in these papers is of variational character and the strong force condition is used to get some compactness for the functional associated with the equation.

Our approach to problem (1.1) is mainly based on degree theoretical methods, and covers both attractive and repulsive potentials together with complete dissipation. We also investigate a conservative case with a repulsive potential. To obtain existence of solutions, we embed (1.1) in a one-parameter family of homotopic equations with solutions for which we can obtain a priori estimates. It follows that a

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