Differential and Integral Equations, Volume 3, Number 6, November 1990, pp. 1127–1138.

ON THE SUM OF TWO MAXIMAL MONOTONE OPERATORS

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1. Introduction and statement of the main results. Let $(\mathcal{H}, ((\cdot,\cdot)))$ be a real Hilbert space and let $\mathcal{A}_i : D(\mathcal{A}_i) \subset \mathcal{H} \to \mathcal{H}$, $i = 1,2$, be two linear m-accretive (or equivalently maximal monotone) operators. Let $\mathcal{A}_{i,\lambda}$ for $\lambda > 0$ denote the Yosida-approximation of $\mathcal{A}_i$, $i = 1,2$. It follows from a general result of Da Prato and Grisvard [10] that if the operators $\mathcal{A}_{1,\lambda}$ and $\mathcal{A}_{2,\mu}$ commute for all λ, $\mu > 0$ (or equivalently for some λ, $\mu > 0$) that $\overline{\mathcal{A}_1 + \mathcal{A}_2}$, the closure of $\mathcal{A}_1 + \mathcal{A}_2$, is m-accretive. In general $\mathcal{A}_1 + \mathcal{A}_2$ is not closed but, as is well-known, if $\mathcal{A}_1$ and $\mathcal{A}_2$ satisfies.

$$\big((\mathcal{A}_{1,\lambda}u, \mathcal{A}_{2,\mu}u)\big) \geq 0 \text{ for all } \lambda, \mu > 0 \text{ and } u \in \mathcal{H} \tag{1.1}$$

then, even if $\mathcal{A}_{1,\lambda}$ and $\mathcal{A}_{2,\mu}$ do not commute, $\mathcal{A}_1 + \mathcal{A}_2$ is m-accretive [4]. In particular if $\mathcal{A}_{1,\lambda}$ and $\mathcal{A}_{2,\mu}$ commute and if $\mathcal{A}_1$ is selfadjoint then condition (1.1) is satisfied. Indeed, one verifies that

$$\begin{aligned}\big((\mathcal{A}_{1,\lambda}u,\, \mathcal{A}_{2,\mu}u)\big) &= \big((\mathcal{A}_{1,\lambda})^{\frac{1}{2}}u,\, (\mathcal{A}_{1,\lambda})^{\frac{1}{2}}\mathcal{A}_{2,\mu}u)\big)\\ &= \big((\mathcal{A}_{1,\lambda})^{\frac{1}{2}}u,\, \mathcal{A}_{2,\mu}(\mathcal{A}_{1,\lambda})^{\frac{1}{2}}u)\big) \geq 0\end{aligned}$$

for λ, $\mu > 0$ and $u \in \mathcal{H}$ [13].

The aim of this paper is to prove a nonlinear version of this result. First we recall that a linear m-accretive operator $\mathcal{A}$ in $\mathcal{H}$ is selfadjoint if and only if it is the subdifferential of a convex function $\Phi : \mathcal{H} \to [0,\infty]$ which is lower semicontinuous (l.s.c) satisfying

$$\Phi(u) = \begin{cases} \frac{1}{2}((\mathcal{A}^{\frac{1}{2}}u, \mathcal{A}^{\frac{1}{2}}u)), & \text{if } u \in D(\mathcal{A}^{\frac{1}{2}})\\ +\infty, & \text{otherwise [3].}\end{cases}$$

We consider the following situation. Let $(\Omega, \mathcal{M}, \nu)$ be a σ-finite measure space and let $(H, (\cdot,\cdot))$ be a real Hilbert space with norm $|\cdot| = (\cdot,\cdot)^{\frac{1}{2}}$. Set $\mathcal{H} = L^2(\Omega, H)$, that is the Hilbert space of H-valued (equivalence classes) Bochner measurable functions $u : \Omega \to H$ satisfying $\int_\Omega |u(\omega)|^2\, d\nu(\omega) < \infty$, with the innerproduct $((u,v)) = \int_\Omega (u(\omega), v(\omega))\, d\nu(\omega)$ for $u, v \in \mathcal{H}$.

Received June 11, 1989.
AMS Subject Classifications: 47H05, 47H20.