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DIRECT METHODS FOR SOME DISTRIBUTED GAMES

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Abstract. A two-person zero-sum game for the system governed by a mildly nonlinear system of two elliptic partial differential equations is considered. Under certain assumptions, the existence of a saddle point is proved; the saddle point is characterized as a unique solution of the associated optimality system, which is then solved, by giving a *constructive* existence theorem.

0. Introduction. We study a two-person zero-sum game for a system governed by an elliptic mildly nonlinear system of partial differential equations. Our motivation, among other things, is the possibility of deriving a constructive analytic proof of the existence of the solution of the associated optimality system. To this end we employ the method of alternating monotone iterations.

The monotone iteration method has been used to study nonlinear elliptic systems in Sattinger [10], when the nonlinear terms are quasimonotone. When the nonlinear terms are not quasimonotone, sequences which are oscillating for each component are constructed by Leung in [8], to analyze the elliptic systems. The procedures are eventually generalized to more elaborate cases in [4] and [5] (see also [9]).

Recently, Stojanovic (see [11–12]) initiated direct study of optimality systems (which happen to be nonquasimonotone elliptic or parabolic) in nonlinear control theory for systems governed by partial differential equations, using methods of alternating monotone iterations. Such systems are, in control theory, traditionally left to formal procedures.

In the present paper, we consider a game problem. Difficulties are twofold. First, proving existence of a saddle point requires some interesting analysis; and second, proving a constructive existence theorem is different from [11], since it requires the introduction of a supersolution.

1. Statement of the problem. Let Ω be a bounded domain of $\mathbb{R}^n$,

$$n \leq 5, \tag{1.1}$$

with $C^{1,1}$ boundary $\partial\Omega$. Denote, for any s, $1 \leq s \leq \infty$,

$$L_{s,+}(\Omega) = \{f \mid f \in L_s(\Omega),\ f \geq 0 \text{ a.e. in } \Omega\}, \tag{1.2}$$

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