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ESTIMATES FROM BELOW FOR THE SOLUTIONS TO A CLASS OF SECOND ORDER EVOLUTION EQUATIONS

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(Submitted by: Peter Hess)

In memory of Zdzislaw Opial

1. Introduction. In a recent paper Alikakos and Rostamian [1] considered evolution equations of the form

$$u'(t) + F'(u(t)) = 0 \quad t > 0, \tag{1.1}$$

$$u(0) = u_0 \tag{1.2}$$

in a Banach space E.

Here $F : E \to \mathbb{R}$ is C^1, convex and F' denotes its Frechet derivative. Assuming that F is homogeneous of degree $p > 2$; i.e.,

$$F(t,x) = t^p F(x) \quad \forall t > 0,\ x \in E \tag{1.3}$$

for some $p > 2$, together with some suitable boundedness and coercivity conditions on F', those authors proved that if u denotes the solution to (1.1)–(1.2), then

$$\|u(t)\| \geq K(u_0)(1+t)^{-1/p-2} \quad t \geq t_0, \tag{1.4}$$

where $K(u_0)$ and t_0 are positive constants depending on u_0.

This paper is concerned with an extension of their result to a class of second order evolution equations of the form

$$u''(t) \in Au(t) \quad t > 0 \tag{1.5}$$

$$u(0) = u_0 \tag{1.6}$$

$$\sup_{t>0} \|u(t)\| < +\infty, \tag{1.7}$$

where $A = \partial F$ is the subdifferential of F, F being a proper ($F \not\equiv +\infty$), lower-semicontinuous (l.s.c.) convex function defined on a real Hilbert space H, satisfying (1.3).

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