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AVERAGING, HOMOTOPY, AND BOUNDED SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. Let $f, g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}^N$ be continuous functions, and assume the mappings $(x,t) \to f(x,t)$, $(x,t) \to g(x,t)$ are almost periodic in t, uniformly for x in compact sets (UAP); see [2]. The average of a uniformly almost periodic function f is defined by

$$\bar{f}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(x,t)\,dt.$$

In this paper the differential equation

$$\dot{u} = f(u,t) + g(u,t) \tag{0.1}$$

will be studied by using the unperturbed averaged equation

$$\dot{u} = \bar{f}(u). \tag{0.2}$$

In this connection the Conley index theory will be applied to certain skew product flows associated with (0.1) and (0.2), an idea introduced in [8] and further developed in [5], [9]. The reader is referred to [1] and [6] for the Conley and more general homotopy index theories, and to [7] for the theory of skew-product flows.

In addition to uniform almost periodicity, it will be assumed that $f(x,t)$ is homogeneous in x of degree p, $0 < p < 1$, and $g(x,t)$ is of lower order in x at infinity. That is, it is assumed that the following hold:

(H1) There is a number p, $0 < p < 1$, such that $f(\lambda x, t) = \lambda^p f(x,t)$ for all $\lambda \geq 0$ and $(x,t) \in \mathbb{R}^N \times \mathbb{R}$.

(H2) Let p be the number given in (H1); then

$$\lim_{|x|\to\infty} g(x,t)/|x|^p = 0.$$

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