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SEMIFLOWS GENERATED BY LIPSCHITZ PERTURBATIONS OF NON-DENSELY DEFINED OPERATORS

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Abstract. A variety of problems in differential equations ((abstract) functional differential equations, age-dependent population models (with and without delay), evolution equations with boundary conditions e.g.) can be written as semilinear Cauchy problems with a Lipschitz perturbation of a closed linear operator which is not densely defined but satisfies the resolvent estimates of the Hille and Yosida theorem. A natural generalized notion of solution is provided by the *integral solutions* in the sense of Da Prato and Sinestrari. We derive a variation of constants formula which allows us to transform the integral solutions of the evolution equation to solutions of an abstract semilinear Volterra integral equation. The latter can be used to find integral solutions to the Cauchy problem; moreover one finds sufficient and necessary conditions for the (forward) invariance of closed convex sets under the solution flow. The solution flow can be shown to form a dynamical system. Conditions for the regularity of the flow in time and initial state are derived. The steady states of the flow are characterized and sufficient conditions for local stability and instability are found. Finally the problems mentioned at the beginning are fitted into the general framework.

Introduction. Semilinear Cauchy problems

$$\frac{d}{dt}u(t) = Au(t) + Fu(t), \quad u(0) = x_0 \tag{1}$$

with F being a Lipschitz perturbation of the generator A of a strongly continuous semigroup are well understood. See, e.g., [36], Section 6.1. Quite often one faces nonlinear Cauchy problems, however, which cannot be split up in this way because a nonlinearity appears in the domain of A though the action of A itself is linear. In this case it is sometimes possible to rewrite the problem as a semilinear problem (1) by removing the nonlinearity from the domain of A and incorporating it into the Lipschitz perturbation F. The price to be paid (according to a universal principle of conservation of difficulty) consists in ending up in a larger Banach space (which is not so bad) and with a linear operator which is not densely defined.

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