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NEW ASPECTS IN BIFURCATION WITH SYMMETRY

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Abstract. This paper presents new results emphasizing connections between bifurcation and group theory rather than group representation theory. It is shown that in every compact real Lie group, possibly finite, which is not a direct product of copies of $\mathbb{Z}_2$, there are nontrivial subgroups independent of the representation whose symmetry must be preserved in some bifurcated solutions of any covariant bifurcation problem, provided that bifurcation occurs with a change in sign of the determinant. This result is complemented by a theorem exhibiting a special feature apparently unnoticed so far of bifurcation in case of $\mathbb{Z}_2^k$-covariance, which is next extended to gradient problems under a more general condition of change of Morse index.

1. Introduction. It comes as no surprise that the literature devoted to bifurcation problems involving symmetry draws from both analysis and group theory. However, it is more accurate to say that it draws from both analysis and group *representation* theory, the two being related through the notion of isotropy subgroup. Isotropy subgroups have been crucial to every work having some connection with bifurcation and symmetry. To give only one example, they are a central theme in the recent volume by Golubitsky, Stewart and Schaeffer [7].

Given a group Γ and a representation R of Γ in $GL(\mathbb{R}^n)$, recall that the isotropy subgroup of $x \in \mathbb{R}^n$ relative to R is the subgroup of $\Gamma : \{\gamma \in \Gamma;\ R_\gamma x = x\}$ (here and everywhere else in this paper, R_γ stands for $R(\gamma)$). This definition makes it clear that isotropy subgroups are strongly representation-dependent.

It is the aim of this paper to show that there are connections between bifurcation problems and group theory that do not refer to isotropy subgroups and are virtually representation-independent. In particular, it will be shown that Lie groups, including finite ones, possess subgroups of a special type, here called intrinsic isotropy subgroups, characterized by a representation-independent property, which play a role in bifurcation problems. To be more precise, consider a bifurcation equation $g(\mu, x) = 0$ with g acting from $\mathbb{R} \times \mathbb{R}^n$ to $\mathbb{R}^n$ with $g(\mu, 0) \equiv 0$ and suppose that $\det D_x g(\mu, 0)$ changes sign as μ crosses 0. Suppose also that $g(\mu, \cdot)$ is covariant under some representation R of a Lie group Γ. Then, Theorem 2.1 of the next section ascertains that whenever $\Gamma' \subset \Gamma$ is an intrinsic isotropy subgroup of Γ, a continuum of Γ'-symmetric nontrivial solutions to $g = 0$ bifurcates from $(0, 0)$.

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