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CONVERGENCE OF A NON CONSERVATIVE FINITE DIFFERENCE SCHEME FOR A SYSTEM OF HYPERBOLIC CONSERVATION LAWS*

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Abstract. We construct a non conservative finite difference approximation to a strictly hyperbolic 2×2 system of conservation laws. The system has one genuinely nonlinear and one linearly degenerate characteristic field. We prove that a subsequence of the family of approximate solutions converges towards a weak solution. We also show that the weak solution generated as the limit of the approximate solutions, satisfies a set of entropy conditions.

1. Introduction. The main purpose of this paper is to prove convergence of approximate solutions generated by a non conservative first order finite difference scheme to a weak solution of the following system of hyperbolic conservation laws:

$$\begin{aligned} u_t + (ug(u,v))_x &= 0 \\ v_t + (vg(u,v))_x &= 0. \end{aligned} \tag{1.1}$$

Here (u, v) denotes the unknown state vector, and g is a given smooth scalar function. The system has one genuinely nonlinear and one linearly degenerate characteristic field. We will consider the Cauchy problem of (1.1) with suitable assumptions on the initial data. First we prove that the linearly degenerate characteristic field does not generate discrete shocks. In fact we prove that one of the Riemann invariants remains Lipschitz continuous. This property is used to prove that if the family of approximate solutions converges boundedly almost everywhere, then the limit is a weak solution of the system. Convergence is verified for a class of functions g for which the system is strictly hyperbolic. We also prove that for this class of functions, the weak solution constructed as the limit of the finite difference scheme, satisfies a set of entropy conditions formulated in terms of the Riemann invariants of the system.

It is a well-accepted hypothesis in general that linearly degenerate fields behave almost linearly in the sense that they do not generate discontinuities, cf. Liu [12]. Here we prove the hypothesis for a special case using finite differences.

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