Differential and Integral Equations, Volume 3, Number 5, September 1990, pp. 909–921.

PERIODIC SOLUTIONS OF HAMILTONIAN SYSTEMS WITH STRONG RESONANCE AT INFINITY

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0. Introduction. In this paper, we study the existence and multiplicity of T-periodic solutions for the Hamiltonian system of second order

$$-x'' = \nabla_x V(t,x) + h(t) \tag{0.1}$$

where $V \in C^1(\mathbb{R} \times \mathbb{R}^K, \mathbb{R})$ is a T-periodic ($T > 0$) function in the variable t and $h \in L^2([0,T], \mathbb{R}^K)$ is such that $\int_0^T h(t)\,dt = 0$. It will be assumed that (0.1) is strongly resonant at infinity (see [4]), i.e.,

(V_1) $V(t,x) \to 0$, when $|x| \to \infty$ (uniformly in $[0,T]$).

(V_2) $\nabla_x V(t,x) \to 0$, when $|x| \to \infty$ (uniformly in $[0,T]$).

Such problems have been studied by many authors. In particular in [1], [7], [14], for $h \equiv 0$, it is proved that if V satisfies (V_1), (V_2) and either

$$\exists r > 0 \,:\, V(t,x) < 0 \quad \forall t \in [0,T],\ \forall x \in \mathbb{R}^K \,:\, |x| \geq r \quad ([1]) \tag{0.2}$$

or

$$\exists \delta > 0,\ \exists \zeta \in \mathbb{R}^K \,:\, V(t,x) < 0 \quad \forall t \in [0,T],\ \forall x \in \mathbb{R}^K \,:\, |\zeta - x| \leq T\sqrt{\frac{M_1}{2}} + \delta$$
$$(M_1 = \sup\{V(t,x) \,/\, (t,x) \in \mathbb{R} \times \mathbb{R}^K\}, [7]) \tag{0.3}$$

or

$$\exists x \in \mathbb{R}^K \,:\, \int_0^T V(t,x)\,dt > 0 \quad ([14]) \tag{0.4}$$

then (0.1) has at least one weak T-periodic solution.

Moreover, in [7], if the author wants to prove existence of a solution for all $h \in L^2([0,T], \mathbb{R}^K)$ with $\int_0^T h(t)\,dt = 0$, he needs, besides (V_1) and (V_2), that $V(t,x) < 0$ for all $(t,x) \in \mathbb{R} \times \mathbb{R}^K$.

Our starting point (inspired by [15]) consists in showing that the conditions (V_1), (V_2) are sufficient for (0.1) to have at least one weak T-periodic solution.

Received April 12, 1989.
AMS Subject Classifications: 34C25, 34B15, 58E05.