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A TRANSFORMATION OF RIGHT-DEFINITE S-HERMITIAN SYSTEMS TO CANONICAL SYSTEMS

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Abstract. It is shown that right-definite S-hermitian boundary value problems (im Normalfall), which were defined and thoroughly studied by Schäfke and Schneider in [5, 6], can be reduced to canonical systems with selfadjoint boundary conditions in such a way that the transformed boundary conditions become a special case of those considered by Dijksma, Langer and de Snoo in [1-3].

1. In this note, the first order system of differential equations

$$F_{11}y' + F_{12}y = \lambda(G_{11}y' + G_{12}y) \tag{1}$$

on the compact interval $I = [a, b]$ is considered. As in [5, 6] we suppose that the $n \times n$-matrix functions F_{1j}, G_{1j} as well as S_{1j} below, $j = 1, 2$, are continuous on I and that $F_{11}(x) - \lambda G_{11}(x)$ is invertible for all $x \in I$ and $\lambda \in \mathbb{R}$. The problem (1) is called S-hermitian (im Normalfall) with respect to the differential operator

$$S_1 y := S_{11}y' + S_{12}y$$

if there exists a continuously differentiable $n \times n$-matrix function H on I such that for all real λ the relationship

$$\begin{pmatrix} F_{11} - \lambda G_{11} & F_{12} - \lambda G_{12} \\ S_{11} & S_{12} \end{pmatrix}^* \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix} \begin{pmatrix} F_{11} - \lambda G_{11} & F_{12} - \lambda G_{12} \\ S_{11} & S_{12} \end{pmatrix} = \begin{pmatrix} 0 & H \\ H & H' \end{pmatrix} \tag{2}$$

holds on I and $H(x)$ is invertible and skew-hermitian for all $x \in I$. If Y denotes the $n \times n$-matrix function which is the solution of the initial value problem

$$F_{11}Y' + F_{12}Y = 0, \quad Y(a) = I_n$$

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