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SPHERICAL MAXIMA IN HILBERT SPACE AND SEMILINEAR ELLIPTIC EIGENVALUE PROBLEMS†

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Introduction. In this paper we study a semilinear eigenvalue problem with Dirichlet boundary condition:

$$\rho A u = f(x,u) \quad \text{on a bounded domain } \Omega \subset \mathbb{R}^n. \tag{1.1}$$

The operator A is a selfadjoint strongly elliptic differential operator of even order 2ℓ with smooth coefficients.

We consider an analog of the first eigenvalue in the linear problem

$$-\Delta u = \lambda u, \quad u \in H_0^1(\Omega). \tag{1.2}$$

Let $\|u\|^2 = \int_\Omega |\nabla u(x)|^2 dx$ and

$$\gamma(t) = \sup_{\|u\|^2 = t} \tfrac{1}{2} \int u(x)^2 dx. \tag{1.3}$$

Then $\gamma(t) = \frac{1}{2}\rho_1 t$, where $\rho_1 = 1/\lambda_1$ and λ_1 is the first eigenvalue of (1.2). The supremum in (1.3) is attained at $u_t = \sqrt{t} u_1$, where u_1 is the first (normalized) eigenfunction in (1.2).

In a similar way we consider $g(u) = \int F(x,u(x))dx$ and

$$\gamma(t) = \sup_{\|u\|^2 = t} g(u), \tag{1.4}$$

where $\|u\|^2$ is a quadratic form of A, equivalent to the Sobolev metric.

It happens that under certain conditions $\gamma(t)$ is a monotone increasing function, the supremum (1.4) is attained and γ has left and right derivatives $\gamma'_\pm$ at every point. These are related to eigenvalues: there exists $u_\pm$, such that $\|u_\pm\|^2 = t$ and

$$2\gamma'_\pm(t) A u_\pm = F'_u(x, u_\pm), \quad \gamma'_\pm(t) \geq 0. \tag{1.5}$$

In Section 2, we provide a preliminary analysis of the function (1.4) in an abstract framework. In Section 3, we apply the results to elliptic operators.

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