Differential and Integral Equations, Volume 3, Number 5, September 1990, pp. 875–887.

GROUND STATES FOR CRITICAL SEMILINEAR SCALAR FIELD EQUATIONS

EZZAT S. NOUSSAIR
School of Mathematics, University of New South Wales
Kensington, N.S.W., Australia 2033

CHARLES A. SWANSON†
Department of Mathematics, University of British Columbia
Vancouver, Canada V6T 1Y4

(Submitted by: A.R. Aftabizadeh)

Abstract. The existence of ground states $u \in L^{\tau+1}(\mathbb{R}^N)$ and asymptotic decay estimates for $u(x)$ as $|x| \to \infty$ are obtained for semilinear scalar field equations involving the critical Sobolev exponent $\tau = (N+2)/(N-2)$, $N \geq 3$. In particular sufficient conditions are given for $u(x) = 0(|x|^{2-N})$ as $|x| \to \infty$.

1. Introduction. Our results concern the existence and asymptotic behavior of positive solutions $u(x)$ in $\mathbb{R}^N$, $N \geq 3$, of semilinear elliptic problems including the prototype

$$\begin{cases} -\Delta u = p(x)u^\tau + q(x)u^\gamma, & x \in \mathbb{R}^N \\ u \in C^2_{loc}(\mathbb{R}^N), & \lim_{|x|\to\infty} u(x) = 0, \end{cases} \tag{1}$$

where τ denotes the critical Sobolev exponent; i.e., $\tau = (N+2)/(N-2)$, and $1 < \gamma < \tau$ if $N \geq 4$, $3 < \gamma < 5$ if $N = 3$. Such solutions $u(x)$ are often called (critical) zero-mass ground states because of their interpretation in quantum field theory [3, 19]. The hypotheses for (1) are as follows: p and q are nontrivial, nonnegative, locally Hölder continuous functions in $\mathbb{R}^N$ such that $p(x) = 0(|x|^{-a})$, $q(x) = 0(|x|^{-b})$ as $|x| \to \infty$ for constants a, b satisfying

$$a > 0, \quad b > \frac{1}{2}[N + 2 - \gamma(N-2)]. \tag{2}$$

This implies in particular that $(N-2)(\gamma-1) > 2(2-b)$ for $0 < b < 2$. In addition there exists a bounded domain $\Omega \subset \mathbb{R}^N$ such that

$$\inf_{x\in\Omega} q(x) > 0, \quad \sup_{x\in\Omega} p(x) = \sup_{x\in\mathbb{R}^N} p(x) \equiv \|p\|_\infty. \tag{3}$$

It is not required that either p or q be radially symmetric.

Received May 10, 1989.
†Supported by NSERC (Canada).
AMS Subject Classifications: 35J60, 35B40, 35Q20.