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ON NEW EXACT BLOW–UP SOLUTIONS FOR NONLINEAR HEAT CONDUCTION EQUATIONS WITH SOURCE AND APPLICATIONS

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Abstract. In this paper, the method of construction of new exact blow-up solutions of quasilinear parabolic equations of nonlinear heat conduction with source are presented. For example, for semilinear equation $u_t = u_{xx} + (1+u)\cdot \ln^2(1+u)$, $t > 0$, $x \in \mathbb{R}^1$, this exact solution has the form $u(t,x) = \exp\{\varphi(t)\cdot[\psi(t)+\cos(x)]\} - 1$, where functions $\varphi(t)$, $\psi(t)$ satisfy the system of ordinary differential equations $(\varphi\psi)' = \varphi^2\cdot\psi^2 + \varphi^2$, $\varphi' = 2\varphi^2\cdot\psi - \varphi$, $t > 0$. Some applications to localization properties of blow-up solutions of degenerate quasilinear parabolic equations

$$u_t = (|u_x|u_x)_x + u^2 \quad \text{and} \quad u_t = (u^\sigma u_x)_x + u^{\sigma+1},$$

where $\sigma > 0$ is a fixed constant, are given.

1. Introduction. In this paper, the method of construction of new exact solutions of quasilinear parabolic equations

$$u_t = u_{xx} + (1+u)\cdot\ln^2(1+u); \tag{1}$$

$$u_t = \nabla\cdot(|\nabla u|\nabla u) + u^2, \quad \nabla(\,\cdot\,) = \text{grad}_x(\,\cdot\,); \tag{2}$$

$$u_t = \nabla\cdot(e^u\nabla u) + e^{-u}\cdot(e^u-\gamma)^2, \quad \gamma = \text{const} > 0; \tag{3}$$

$$u_t = (u^\sigma\cdot u_x)_x + u^{1-\sigma}\cdot(u^\sigma-\gamma)^2, \quad \sigma = \text{const} > 0; \tag{4}$$

$$u_t = (u^{-1}u_x)_x + \delta u^2, \quad u_t = (u^{-1}u_x)_x + (u^{-1}u_y)_y + \delta u^2, \quad \delta = \text{const} \neq 0, \tag{5}$$

and some generalizations are presented.

It is well known that the effective method for investigation of asymptotic properties of solutions of quasilinear evolution equations consists in construction suitable self-similar or invariant solutions which are determined from ordinary differential equations. These particular exact solutions have a simple space-time structure which determines much of the required information regarding the asymptotic development of the nonlinear process. In many cases exact solutions play an important

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