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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO A CLASS OF NONLINEAR VOLTERRA EQUATIONS

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Abstract. The asymptotic properties of solutions to a nonlinear Volterra integral equation are studied in a general Banach space. The concept of a completely positive kernel plays a crucial role in the analysis.

1. Introduction. The purpose of this paper is to discuss the asymptotic behavior as $t \to \infty$ of solutions to the abstract Volterra equation

$$u(t) + \int_0^t b(t-s)\big(Au(s) + g(s)u(s)\big)\,ds \ni f(t), \quad t \in \mathbb{R}^+ = [0,\infty), \qquad (V_{b,g,f})$$

in a real Banach space X. Here $b : \mathbb{R}^+ \to \mathbb{R}^+$ is a completely positive kernel, A is a nonlinear (possibly multivalued) m-accretive operator in X, $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a given function, f maps $\mathbb{R}^+$ into X, and the integral is taken in the sense of Bochner.

General existence, uniqueness and continuous dependence results for $(V_{b,g,f})$ have been established by Crandall and Nohel [8] and Gripenberg [10]. The asymptotic properties of solutions of $(V_{b,g,f})$ have primarily been studied in the case where $g \equiv 0$. See, e.g., [2, 4, 5, 13, 16]. Recently, Kato, Kobayasi and Miyadera [15] have discussed the asymptotic behavior of solutions to a class of functional-differential equations related to $(V_{b,g,f})$. When applied to $(V_{b,g,f})$, their theory requires that $0 \in R(A)$ and $g \in L^1(\mathbb{R}^+)$, being thereby restricted to bounded solutions.

The present work is mainly concerned with the "unbounded behavior," as $t \to \infty$, of solutions to $(V_{b,g,f})$, so that we generally assume that $R(A)$ is zero free and $g \notin L^1(\mathbb{R}^+)$. Our study can be viewed as an attempt to extend earlier results obtained by Israel and Reich [14], and Kobayasi [17] for $(V_{1,g,f})$ (that is, the case where $(V_{b,g,f})$ reduces to an evolution equation), as well as the asymptotic theory

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