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REMARKS ON THE DAMPED STATIONARY EULER EQUATIONS

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Abstract. We consider the Charney-Stommel model of the Gulf-Stream, taking into account the Coriolis force and the friction due to the bottom of the ocean. It reduces to a damped version of the stationary Euler equations in space dimension 2. We prove the existence of weak solutions for any body force in H^1, and the existence, uniqueness and stability of smooth solutions when the force is small relative to the damping.

Introduction. This paper[1] is concerned with a system arising in the theory of ocean circulation. The system under study is due to Stommel [15] and Charney [5] (see also the book of Pedlosky [12]). It is a model of the Gulf-Stream taking into account the Coriolis force (the so-called β-effect) and the friction due to the bottom of the ocean. Mathematically, it reduces to a boundary value problem for the damped stationary Euler equations of incompressible fluids. More precisely, let Ω be a bounded simply connected domain of $\mathbb{R}^2$ with smooth boundary Γ. We are looking for a vector field $u = (u_1, u_2)$ and a scalar pressure p which satisfy the system

$$\begin{cases} \epsilon u + R(u.\nabla)u + \nabla p + \varphi = f & \text{in } \Omega \\ div\, u = 0 & \text{in } \Omega \\ u.n|_\Gamma = 0. \end{cases} \tag{0.1}$$

Here $\epsilon > 0$ and $R \geq 0$ are given and arise as a result of the nondimensionalization of the various fields; f is a driving force due to the applied surface wind stress; the term φ is due to the Coriolis force and plays to no major mathematical role in our analysis. It is related to u by

$$\underline{curl}\, \varphi = u_2. \tag{0.2}$$

We will determine it uniquely by imposing

$$div\, \varphi = 0 \quad \varphi.n|_\Gamma = 0 \tag{0.3}$$

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