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A MIXED TYPE BOUNDARY PROBLEM DESCRIBING THE PROPAGATION OF DISTURBANCES IN VISCOUS MEDIA

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Abstract. In this paper the following mixed type problem

$$u_{tt} = Au_t + b(x,t,u,u_x,u_{xx},u_{xt},u_t)$$

with certain initial and boundary conditions in n-dimensional space is studied, where A is a general elliptic linear operator. The existence, uniqueness and continuous dependence of the solution are demonstrated under the appropriate assumptions.

1. Introduction. In this paper we consider the following mixed type problem

$$u_{tt} = Au_t + b(x,t,u,u_x,u_{xx},u_{xt},u_t), \quad \text{in } Q_T, \tag{1.1}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \tag{1.2}$$

$$u(x,t) = \psi(x,t), \quad \text{on } S_T = \partial\Omega \times [0,T], \tag{1.3}$$

where $Q_T = \Omega \times (0,T]$, $T > 0$, Ω is an open bounded region in R^n with $\partial\Omega \in H^{2+\alpha}$ $(0 < \alpha < 1)$ and

$$Au = \sum_{i,j=1}^{n} a_{ij}(x,t)u_{x_i x_j}(x,t),$$

and where a_{ij}, u_0, u_1, ψ and b are known functions. Here and throughout the paper we shall use the standard notations defined in Chapter 1 of [10], $u_x = (u_{x_1}, \cdots, u_{x_n}) \in R^n$ and $u_{xx} = (u_{x_1x_1}, u_{x_1,x_2}, \cdots, u_{x_nx_n}) \in R^{n^2}$.

The motivation for studying (1.1)-(1.3) comes from the propagation of disturbances in viscous media which can be described by the equation

$$u_{tt} = cu_{xxt} + u_{xx},$$

where $c > 0$ is a constant and cu_{xxt} represents the perturbing effects of a small viscosity.

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