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AN EXPLICIT CLOSED FORM SOLUTION FOR SYSTEMS OF VOLTERRA INTEGRAL EQUATIONS*

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Abstract. In this paper, we propose a method for the solution of systems of Volterra integral equations from an algebraic point of view. An explicit finite closed form solution of the problem in terms of co-solutions of a certain algebraic matrix equation is given.

1. Introduction. Volterra integral equations appear in many applications in system theory as system identification [12], system modeling [4], and damped vibrations [11]. Systems of Volterra equations appear, for example, in population competition [3]. Systems of Volterra equations of the type

$$\Psi_i(x) + \int_0^x \Big\{ \sum_{j=1}^{n-1} b_{ij} \Psi_j(t)(x-t)^j/j! \Big\} dt = f_i(x), \quad 1 \le i \le n, \tag{1.1}$$

where b_{ij} are complex numbers for $1 \le j \le n-1$, and $f_i : [0,\infty[\to \mathbb{C}$, is continuous for $1 \le i \le n$, are well studied from a classical point of view. So, the Picard theorem [12, p. 62], provides the unique continuous solution of system (1.1) as the limit of a sequence of successive approximations. Numerical methods for solving systems of Volterra equations may be found in [1, 7, 9, 12, 13].

Note that system (1.1) may be written in the compact form

$$\Psi(x) + \int_0^x \{B_1 + B_2(x-t) + \cdots + B_n(x-t)^{n-1}/(n-1)!\}\, \Psi(t)\, dt = f(x), \tag{1.2}$$

where

$$\Psi = \begin{bmatrix} \Psi_1 \\ \vdots \\ \Psi_n \end{bmatrix}, \quad f = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}, \quad B_j = \left(b^j_{pq}\right), \ b^j_{pj} = b_{pj},\ b^j_{pq} = 0,\ q \ne j,\ 1 \le i, j \le n.$$

The aim of this paper is to find an explicit closed form solution of systems of Volterra equations of the type (1.2) where $f : [0,\infty[\to \mathbb{C}^n$ is continuous and B_i, $1 \le i \le n$, are arbitrary matrices in $\mathbb{C}^{n\times n}$.

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