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EXPONENTIAL ASYMPTOTIC STABILITY FOR SCALAR LINEAR VOLTERRA EQUATIONS

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Abstract. Exponential asymptotic stability of the zero solution of the scalar linear Volterra equation

$$\dot{x}(t) = Ax(t) + \int_0^t B(t-s)x(s)\,ds$$

is studied. Roughly speaking, it is proved that the exponential asymptotic stability can be characterized by a growth condition on $B(t)$. The result answers the problem posed by Corduneanu and Lakshmikantham.

In this article, we shall be concerned with a linear Volterra equation with an integrable kernel B,

$$\dot{x}(t) = Ax(t) + \int_0^t B(t-s)x(s)\,ds \quad t \geq 0, \tag{E}$$

and study the exponential asymptotic stability of the zero solution of (E) in conjunction with the exponential behavior of $|B(t)|$ as $t \to \infty$. The subject deeply relates to the paper [1] due to Corduneanu and Lakshmikantham. In fact, they have posed the following problem [1, pp. 845–848]: If the zero solution of (E) is uniformly asymptotically stable, then is it of exponential type? To analyze the problem, we shall focus our attention to the case where (E) is a scalar equation, and show (Theorem 1) that if B satisfies a growth condition, then the above problem can be solved in the affirmative. Furthermore, under the restriction on B, we shall investigate the converse of our Theorem 1, too. In fact, under the assumption that $B(t)$ does not change sign on $[0,\infty)$, we prove (Theorem 2) that if the zero solution of (E) possesses the stability property of exponential type, then B satisfies a growth condition. Thus (roughly speaking) the stability of exponential type for the zero solution of (E) can be characterized by a growth condition on B (Theorem 3). Hence, the stability of exponential type can never be realized for the equation with a kernel B which does not satisfy a growth condition, and the problem posed above can be solved in the negative.

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