Differential and Integral Equations, Volume 4, Number 3, May 1991, pp. 493–518.

BOUNDED SOLUTIONS FOR ABSTRACT TIME-PERIODIC PARABOLIC EQUATIONS WITH NONCONSTANT DOMAINS

M. FUHRMAN
Dipartimento di Matematica, Università degli studi di Milano
Via C. Saldini 50, 20133 Milano, Italy

(Submitted by: G. Da Prato)

Abstract. We consider linear nonautonomous parabolic equations in Banach spaces associated with a family of operators $\{A(t);\ t \in \mathbb{R}\}$ with nondense time-varying domains, generating analytic semigroups. Under the periodicity assumption $A(t+T) = A(t)$ we study existence, regularity and asymptotic properties of bounded and periodic solutions in unbounded time intervals.

1. Introduction. We are going to study the abstract linear evolution equation with unknown u

$$\begin{cases} u'(t) = A(t)u(t) + f(t), & t \in I \\ u(t_0) = x, \end{cases} \tag{1.1}$$

where $I \subset \mathbb{R}$ is an unbounded interval, $t_0 \in \mathbb{R}$, u and f are functions from I to a Banach space E and $x \in E$, $\{A(t)\}_{t\in\mathbb{R}}$ is a family of closed linear operators in E with domains $D_{A(t)}$ that may be nondense in E and may vary with t. We assume that each $A(t)$ generates a bounded analytic semigroup in E, in the sense of [12], and we make some continuity assumptions on the dependence of the resolvent of $A(t)$ on t. Our hypotheses are the same as in [3], [4], [1]. In this situation an evolution operator $U(t,s)$ for the family $\{A(t)\}$ can be defined (see [4], [1]).

We consider the periodic case:

$$A(t+T) = A(t).$$

We are therefore interested in a generalization of Floquet's theory to abstract linear equations (see [7]). We state conditions for existence and uniqueness of bounded and periodic solutions of problem (1.1). We also find regularity properties of the solutions, we give a representation formula for them and we study their asymptotic behavior. Here is a sample of our results (in a rough form):

Suppose $x \in D_{A(t_0)}$, $A(t_0)x + f(t_0) \in D_{A(t_0)}$ and f is Hölder continuous and bounded. Suppose the spectrum of $U(t+T,t)$ does not intersect the unit circle for any $t \in \mathbb{R}$ ($U(t,s)$ is the evolution operator associated with $\{A(t)\}_{t\in\mathbb{R}}$; see Section

Received for publication April 17, 1990.
AMS Subject Classifications: 34G10, 35K22.