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A NUMERICAL METHOD FOR NONLINEAR AGE DEPENDENT POPULATION MODELS

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Abstract. For hyperbolic equations, if the characteristics are known, then the solution can be obtained by tracing back the characteristics. In this study, a numerical method is developed for nonlinear age-dependent population models by properties of hyperbolic type of equations. The error estimates for the method as well as some numerical experiments are presented.

1. Introduction. Mathematical models of population dynamics have been playing important roles in the study of various kinds of populations, from the population of cells to the population of human species. The simplest population model was proposed by T.R. Malthus in 1798. Forty years later, P.T. Verhulst modified Malthus' model by considering the effects of crowding and the limitation of resources. Since then the theory of population dynamics has been developed by many researchers [5], [9], [11]. Among many significant results, the age-dependent population models are particularly of interest in this field [5], [9], [11], because they are closer to the real situation. Moreover, they brought the PDE theories into the analysis of population dynamics. The first nonlinear age-dependent population model was proposed by M.E. Gurtin, R.C. MacCamy and F. Hoppensteadt in 1974 [2-4]. They allowed the mortality rate and the fertility rate to be affected by the total population, which is true for most real cases. There is considerable literature on the analysis of population models. Some recent results are due to [1], [8], [12] and [13].

Because of the practical applications of population models, the numerical approach to the problem is also very important. There are several numerical methods proposed for treating the problem [6-9], [11], [13]. A typical idea to treat hyperbolic type of linear age-dependent population models is to discretize the age variable, so that the PDE is changed to an ODE system. However, solving the ODE system explicitly requires a very strong stability condition, i.e., the time step Δt has to be extremely small [6]. This is not practical. On the other hand, to solve the system by an *implicit* scheme involves inversion of matrices which may significantly increase the computation cost. Moreover, it is very difficult to implement the implicit scheme if the model is nonlinear. In Section 2 of this paper, a new numerical algorithm is given based on the fact that the characteristics of the age-dependent population models are simply straight lines. The algorithm is a fully explicit scheme which is applicable to both linear and nonlinear cases. Only function values at discretized

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